FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r .

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Calculus Unit Practice

1. **a)** Find $f'(x)$, given that $f(x) = 5\sqrt{x} - \frac{2}{x^3}, x > 0$.

b) Find $f'(x)$, given that $f(x) = 2\sqrt{x} + 3x^{-4}, x > 0$.

c) Find $f'(x)$, given that $f(x) = 2x^{\frac{1}{2}} - \frac{3}{x^5}, x > 0$.

d) Find $f'(x)$, given that $f(x) = 6\sqrt{x} - \frac{5}{x^6}, x > 0$.

2 **a)** Differentiate the function $f(x) = 4cosx$ with respect to *x*.

b) Differentiate the function $f(x) = 7sinx$ with respect to *x*.

c) Differentiate the function $f(x) = -2cosx$ with respect to *x*.

d) Differentiate the function $f(x) = 3\cos x$ with respect to *x*.

3.a) A curve has equation $y = 3x^2 + 2x + 2$, find the equation of the tangent to the curve at $x = -1$.

b) A curve has equation $y = 5x^2 - 3x + 2$, find the equation of the tangent to the curve at $x = 2$.

c) A curve has equation $y = 4x^2 + 2x - 1$, find the equation of the tangent to the curve at $x = -2$.

d) A curve has equation $y = 3x^2 - 2x + 5$, find the equation of the tangent to the curve at $x = 1$.

4a) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 40t - 2t^2.$

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 10 seconds after it is set off.

b) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 30t - t^2.$

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 15 seconds after it is set off.

c) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 60t - 6t^2.$

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 5 seconds after it is set off.

d) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 16t - 4t^2.$

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 2 seconds after it is set off.

5.a) Find $\int 5x^{\frac{3}{2}} + \frac{1}{x^3}\ dx, x \neq 0.$

b) Find $\int 2x^{\frac{1}{2}} - \frac{1}{x^5}\ dx, x \neq 0.$

c) Find $\int 4x^{\frac{2}{3}} + \frac{1}{x^2}\ dx, x \neq 0.$

d) Find $\int 5x^{\frac{1}{4}} - \frac{1}{x^7}\ dx, x \neq 0.$

6. a) $f'(x) = (x+3)^{-7}$, find f(x), x ≠-3.

b) $f'(x) = (x-1)^{-6}$, find f(x), x ≠1.

c) $f'(x) = (x+4)^{-3}$, find f(x), x ≠-4.

d) $f'(x) = (x-9)^{-2}$, find f(x), x ≠9.

7. a) Find $\int 3\cos\theta \, d\theta$

b) Find $\int 2\sin\theta \, d\theta$

c) Find $\int -6\cos\theta \, d\theta$

d) Find $\int 4\cos\theta \, d\theta$

8.a) $\int_1^3 (x+1)^3$

b) $\int_1^2 (x-5)^4$

c) $\int_2^3 (x+2)^6$

d) $\int_1^4 (x-7)^2$

9. (a) A box with a square base and open top has a surface area of 192cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 48x - \frac{1}{4}x^3 \ where \ x > 0$$

Find the value of x which maximises the volume of the box.

(b) A box with a square base and open top has a surface area of 972cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 243x - \frac{1}{4}x^3 \ where \ x > 0$$

Find the value of x which maximises the volume of the box.

(c) A box with a square base and open top has a surface area of 432cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 108x - \frac{1}{4}x^3 \ where \ x > 0$$

Find the value of x which maximises the volume of the box.

(d) A box with a square base and open top has a surface area of 484cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 121x - \frac{1}{4}x^3 \ where \ x > 0$$

Find the value of x which maximises the volume of the box.

10. (a) The curve with equation $y = x^2(3 - x)$ is shown below.

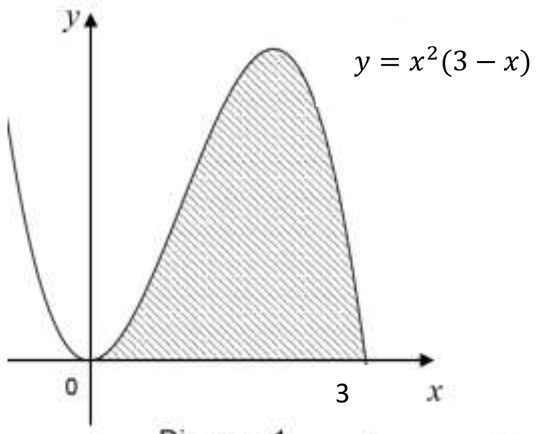


Diagram 1

5

Calculate the shaded area.

(b) The curve with equation $y = x^2(5 - x)$ is shown below.

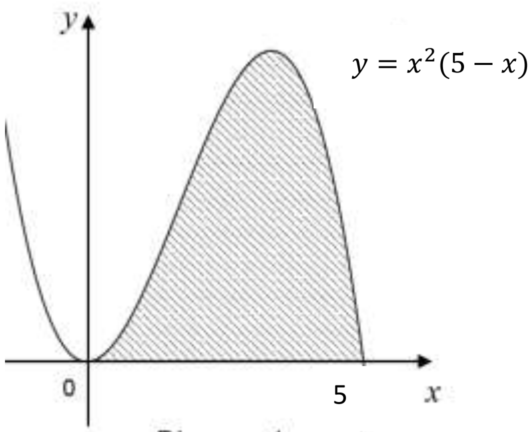


Diagram 1

5

Calculate the shaded area.

(c) The curve with equation $y = x^2(6 - x)$ is shown below.

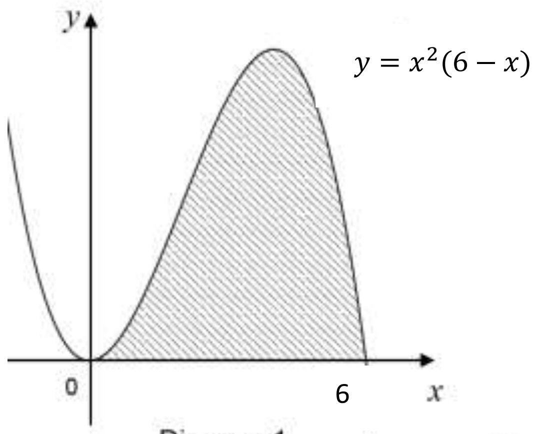


Diagram 1

5

Calculate the shaded area.

(d) The curve with equation $y = x^2(10 - x)$ is shown below

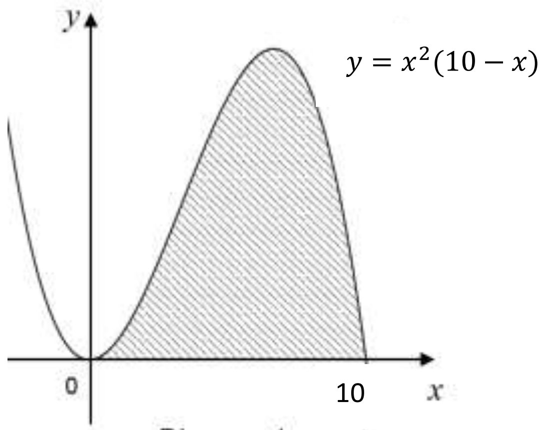


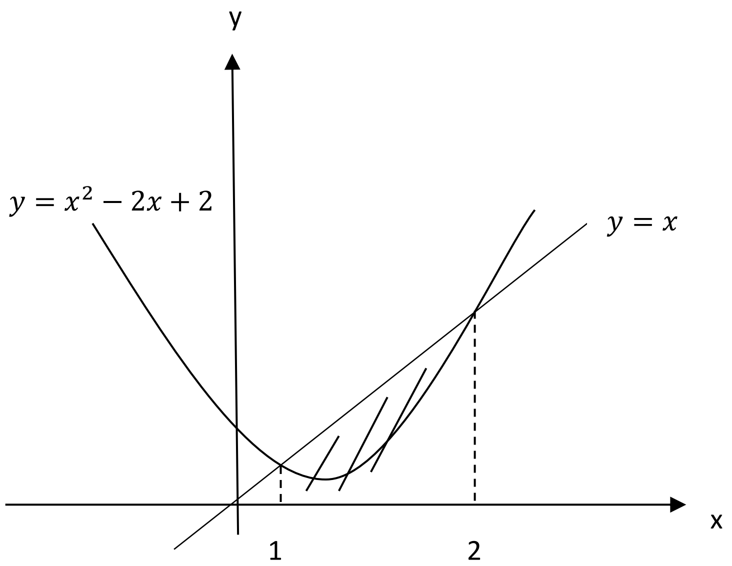
Diagram 1

5

Calculate the shaded area.

11. (a) The line with equation $y = x$ and the curve with equation

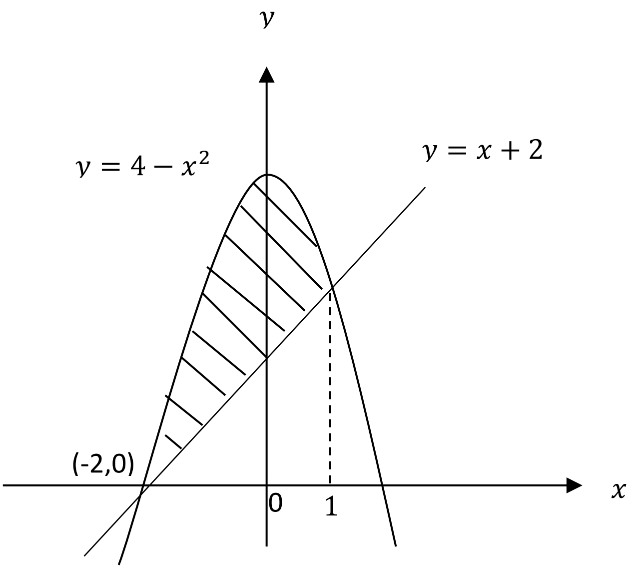
$y = x^2 - 2x + 2$ are shown below



The line and the curve meet at the points where $x = 1$ *and* $x = 2$.

Calculate the shaded area.

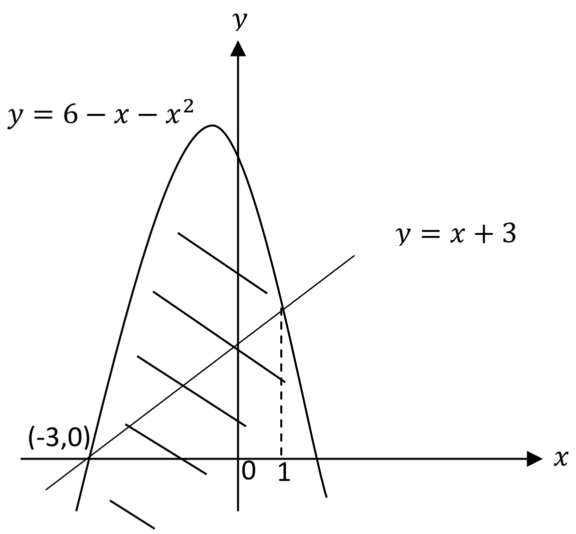
(b) The line with equation $y = x + 2$ and the curve with the equation $y = 4 - x^2$ are shown below.



The line and the curve meet at the points where $x = -2$ and $x = 1$.

Calculate the shaded area.

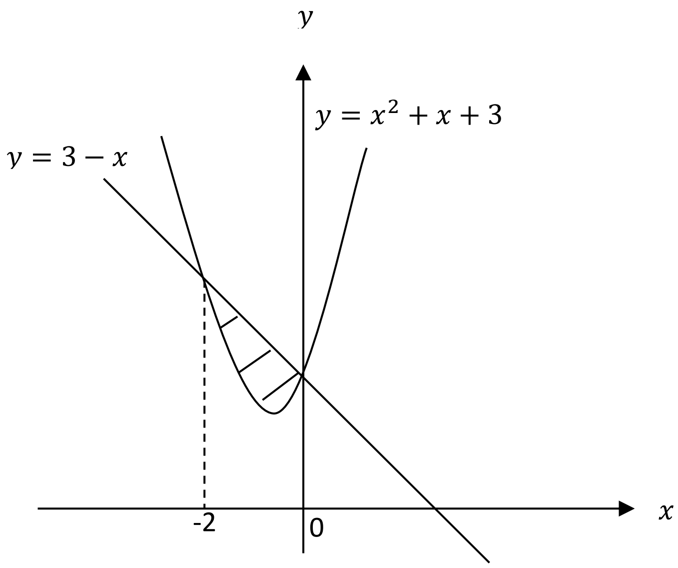
(c) The line with equation $y = x + 3$ and the curve with equation $y = 6 - x - x^2$ are shown below.



The line and the curve meet at the points where $x = -3$ and $x = 1$.

Calculate the shaded area.

(d) The line with equation $y = 3 - x$and the curve with equation $y = x^2 + x + 3$ are shown below.



The line and the curve meet at the points where $x = -2$ and $x = 0$.

Calculate the shaded area.

Answers

1. **a)** $f'(x) = \frac{5}{2}x^{-\frac{1}{2}} + 6x^{-4}$ **b)** $f'(x) = x^{-\frac{1}{2}} - 12x^{-5}$

c) $f'(x) = x^{-\frac{1}{2}} + 15x^{-6}$ **d)** $f'(x) = 3x^{-\frac{1}{2}} + 30x^{-7}$

2. **a)** $f'(x) = -4sinx$ **b)** $f'(x) = 7cosx$

c) $f'(x) = 2sinx$ **d)** $f'(x) = -3sinx$

3.a) $y - 3 = -4(x + 1)$ **b)** $y - 16 = 17(x - 2)$

c) $y - 11 = -14(x + 2)$ **d)** $y - 6 = 4(x - 1)$

4. **a)** **i)** $v = 0$ **ii)** The flare has stopped rising

b) **i)** $v = 0$ **ii)** The flare has stopped rising

c) **i)** $v = 0$ **ii)** The flare has stopped rising

d) **i)** $v = 0$ **ii)** The flare has stopped rising

5. a) $2x^{\frac{5}{2}} - \frac{1}{2}x^{-2} + c$ **b)** $\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^{-4} + c$

c) $\frac{12}{5}x^{\frac{5}{3}} - x^{-1} + c$ **d)** $4x^{\frac{5}{4}} + \frac{1}{6}x^{-6} + c$

6. **a)** $f'(x) = -\frac{1}{6}(x + 3)^{-6} + c$ **b)** $f'(x) = -\frac{1}{5}(x - 1)^{-5} + c$

c) $f'(x) = -\frac{1}{2}(x + 4)^{-2} + c$ **d)** $f'(x) = -(x - 9)^{-1} + c$

7. **a)** $3sin\boldsymbol{\theta} + \boldsymbol{c}$ **b)** $-2cos\boldsymbol{\theta} + \boldsymbol{c}$ **c)** $-6sin\boldsymbol{\theta} + \boldsymbol{c}$ **d)** $4sin\boldsymbol{\theta} + \boldsymbol{c}$

8. **a)** 60 **b)** $\frac{781}{5}$ **c)** $\frac{61741}{7}$ d) 63

9 (a) Max at $x = 8$

(b) Max at $x = 18$

(c) Max at $x = 12$

(d) Max at $x = 12 \cdot 7$

10 (a) $6\frac{3}{4}$ (b) $52\frac{1}{12}$ (c) 108 (d) $833\frac{1}{3}$

11 (a) $\frac{1}{6}$ (b) $4\frac{1}{2}$ (c) $10\frac{2}{3}$ (d) $\frac{4}{3}$