Maths Higher – Prelim Content

Straight Line

Gradient of a line $A(x_1, y_1), B(x_{2,}y_2)$, Gradient of AB $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$m = tan\theta$ where θ is the angle the line makes with the positive direction of the x-axis.

If the gradient, m, is positive then θ is acute. If the gradient, m, is negative then θ is obtuse.

Perpendicular (at right angles) Lines:

If two lines with gradients m_1 and m_2 are perpendicular then $m_1 \times m_2 = -1$

Midpoint:

The midpoint of line AB where A is (x_1, y_1) and B is (x_2, y_2) is found by Midpoint = $(\frac{x_{1+}x_{2,}}{2}, \frac{x_2+y_2}{2})$

Equation of a line:

(1) $y = mx + c$

(2) $y - b = m(x - a)$

Eliminate fractions, multiply out brackets and put in form if you need to use simultaneous equations.

(3) $ax + by + c = 0$

Rearrange into form (1) if you need to find the gradient or y-intercept.

Intersection with the x-axis: Make y=0 and solve. Intersection with the y-axis. Make x=0 and solve.

Types of Line

Perpendicular Bisector – Line through the midpoint, perpendicular to the given line.

Method: Find gradient of line, find perpendicular gradient, find midpoint. Write equation using perpendicular gradient and midpoint.

Median – Line from a vertex (corner) of a triangle to the midpoint of the opposite side.

Method: Find midpoint of opposite side, find the gradient using the vertex and midpoint, state the equation using gradient and either vertex or midpoint.

Altitude – Line from a vertex of a triangle at right angles to the opposite side.

Method: Find the gradient of the opposite side, find perpendicular gradient, state the equation using the perpendicular gradient and the vertex.

Point of Intersection of Two lines

To find the coordinates of the point of intersection of two lines use **simultaneous equations.**

Circle

Equations: $(x-a)^2+(y-b)^2=r^2$ has centre (a,b) and radius r.

$x^2+y^2+2gx+2fy+c=0$ has centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$.

Both these formulae are on the formulae sheet.

Tangent to a circle: To show a line is a tangent to a circle substitute the equation of the tangent into the circle equation and show there is only one point of contact by solving or using $b^2-4ac=0$, and **stating** that since equal roots the line is a tangent.

To find the equation of a tangent to a circle at Point P(x,y). Find the centre, C, of the circle. Find the gradient of the radius CP. Find the gradient of the tangent (perp. Gradients) and state the equation of the tangent.

Intersection of Circles: You must be able to find the centre and radii of circles (r_1, r_2) and hence the distance, d, between their centres to see whether the circles

1) Intersect in two places d < (r_1+r_2)
2) Intersect in one place (touch externally) d = (r_1+r_2)
3) No points of intersection d > (r_1+r_2). Find the smallest gap between the circles.

You can also use the centres and radii to show that circles touch internally.

VECTORS

For a point P(x,y,z), Position vector $\overrightarrow{OP}=\begin{pmatrix}x\\y\\z\end{pmatrix}$.

Vector $\overrightarrow{AB}=b-a$.

i, j and k are unit vectors in the x, y and z directions respectively.

Collinearity:

To show that points A, B and C are collinear, find vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Show that $\overrightarrow{AB}$=p$\begin{pmatrix}x\\y\\z\end{pmatrix}$

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$\overrightarrow{BC}=q\begin{pmatrix}x\\y\\z\end{pmatrix}$, *this means that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel and since B is a common point then*

A, B and C are collinear. You **must** include all of the above in your answer.

The ratio that B divides AC will be p:q.

Section Formula:

If *P divides $\overrightarrow{AB}$ in the ratio $m{:}n$ then P is found using the formula* $P=\frac{nA+mB}{m+n}$.

Scalar Product

$a.b = |a||b|cos\theta$ $a.b = a_1b_1 + a_2b_2 + a_3b_3$ (both on formulae sheet)

If vectors are perpendicular (at right angles) then $a \,.\, b = 0$.

The angle between vectors is found using the formula $cos\theta = \frac{a\,.b}{|a||b|} = \frac{a_1b_1+a_2b_2+a_3b_3}{|a||b|}$ (not on formula sheet) but can be found by equating the two vectors formulae given.

Remember that both vectors $a\ and\ b$ must be pointing away from the vertex.

You might also need to use: $a.(b + c) = a.b + a.c$

Vector Pathways

You will be expected to use vectors to describe a pathway along a 3D object.

Functions

You will be expected to understand the terms domain and range and answer problems relating to these terms.

Composite Functions: $f(g(x))$, insert whatever $g(x)$is into f brackets and then replace all x's

in $f(x)$ with $g(x)$.

Find the **inverse function** f^{-1}(x). Make x the subject of the formula, swap x and y and then change y to f^{-1}(x). Remember that $f(f^{-1}$(x)) = x. (This is always true).

Graph Transformations

$f(x) \pm a$ moves $f(x)$ up or down a units. $f(x \pm a)$ moves $f(x)$ left/right by a units.

k ($f(x)$) stretches $f(x)$ vertically by a factor of k. All y values are multiplied by k.

f(kx) compresses f(x) horizontally by a factor of k. Divide all x values by k.

-f(x) reflects f(x) in the x axis. All y coordinates will change from positive to negative and vice-versa.

f(-x)reflects f(x) in the y-axis. All x coordinates will change from positive to negative and vice-versa.

f^{-1}(x) reflects f(x) in the line y=x. (x,y) becomes (y,x).

Polynomials $f(x) = ax^3 + bx^2 + cx + d$ or $y = ax^3 + bx^2 + cx + d$

Evaluate polynomials at a given x value, e.g a, by evaluating $f(a)$.

Remainder/Factor Theorems using the nested form (table). Any missing powers of x must be represented by a 0 in your table.

If $f(x) = ax^3 + bx^2 + cx + d$ is divided by $(x - p)$ and the remainder is = 0 ($f(p) = 0$) then $(x - p)$ is a factor of $f(x)$ and we can use the quotient to fully factorise $f(x)$. Remember to state all the factors including $(x - p)$. **You must state** that Remainder = 0 so $(x - p)$ is a factor.

We can solve a polynomial, $f(x) = 0$, by factorising first and then solving each factor = 0. These are the roots of the equation, where the graph cuts the x-axis.

You must be able to solve polynomial problems involving factors, roots, remainders and unknown coefficients.

You must also be able to sketch a polynomial given its roots and the y-intercept. A repeated root means it is also a turning point.

Quadratics

Completing the square

Express a quadratic function of the form $f(x) = ax^2 + bx + c$ in the form $f(x) = a(x + p)^2 + q$.

This function will have a turning point of $(-p, q)$.

Discriminant $b^2 - 4ac$

Determine the nature of the roots of a Quadratic. You may need to rearrange quadratic first into the form $ax^2 + bx + c = 0$

$b^2 - 4ac$ >0 Real and distinct, $b^2 - 4ac$ = 0 Real and equal, $b^2 - 4ac < 0$ no real roots should be the communication given.

You will be expected to find unknown coefficients in discriminant problems.

To solve a quadratic inequality, e.g. Find when a quadratic has real roots $b^2 - 4ac \geq 0$, make a sketch of the quadratic by finding the roots and use your graph to see when the parabola is on or above the x-axis.

Recurrence Relations

The formula for a Recurrence Relation (RR) is given by $U_{n+1} = aU_n + b$. You will be expected to use this formula to find successive terms of a RR both with and without a calculator.

This sequence will have a Limit (sequence is convergent) if, $-1 < a < 1$, and the Limit is found using the formula $L = \frac{b}{1-a}$.

You will also be expected to solve problems where you are given the Limit and have to find the values of a, b or both depending on the type of question.

Logarithms and Exponentials

Laws of logs

Adding logs of the same base: $log_a x + log_a y = log_a(xy)$.

Subtracting logs of the same base: $log_a x - log_a y = log_a\left(\frac{x}{y}\right)$.

Log of a number raised to a power: $log_a x^n = n log_a x$.

On your calculator **log is $\boldsymbol{log_{10}}$** and **ln is $\mathbf{log_e}$.**

Log statement to Exponential statement

You must be able to change from a log statement to an exponential statement e.g.

$x = log_a y \Leftrightarrow y = a^x$. A numerical example of this would be $2 = log_{10}100 \Leftrightarrow 100=10^2$.

You will be expected to solve log equations using the laws and by changing from Logs to Exponential.

Growth and Decay problems of the type $A_t = A_0 e^{kt}$. For half-life questions $\frac{A_t}{A_0} = 0.5$

The number e may be replaced with another number for this type of question.

Experimental Data

For a relationship of the form $y = ax^b$ the graph of $\mathbf{log}\,\boldsymbol{y}$ ***against*** $\mathbf{log}\,\boldsymbol{x}$ will be a straight line.

For a relationship of the form $y = ab^x$ the graph of $\mathbf{log}\,\boldsymbol{y}$ ***against*** $\boldsymbol{x}$ will be a straight line.

We solve these equations by taking logs of both sides and expressing as a linear equation.

Differentiation

We differentiate to find the derivative which represents the gradient or rate of change of a function.

Notation for differentiation is $\frac{dy}{dx}$ or $f'(x)$.

Often we need to rearrange a function to a series of powers of x before differentiating. We do this by multiplying out brackets or putting a quotient(division) into separate fractions and using our division rule of indices.

Tangent to a curve

The gradient of a curve is equal to the gradient of the tangent to the curve at the point of contact.

Find the derivative of the curve to get the gradient at a given x-value, or, equate the derivative to the gradient if the gradient is known to find the x-value. Substitute your x-value into the equation of the curve