

Numeracy Across the Curriculum

A Guide for Parents, Carers &
Teachers of All Subjects

The Nicolson Institute

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Introduction

Aims of this booklet:

- To enable all parents and carers to adopt a common approach to helping their children with numeracy.
- To provide materials which will support you when assisting with numeracy related homework tasks
- This will help pupils as they will more easily recognize the numeracy skills required for their work and will ensure consistency in the methods they will use.

How to Use this Booklet:

- It is envisaged that parents will 'dip into' the resources as and when necessary.
- The contents page can be used to find materials by topic and there is also a table in the back of the booklet to help you find the materials by the experience / outcome.
- The booklet also highlights how 'not' to teach some aspects of numeracy and some of the language / terms which are no longer used.
- There are also hyperlinks you can use to get to specific pages quickly. To follow them hold down the 'CTRL' key and then 'left click' with the mouse.

Estimation and rounding

Experiences & Outcomes:

Second Level	Third Level	Fourth Level
I can use my knowledge of rounding to routinely estimate the answer to a problem, then after calculating, decide if my answer is reasonable, sharing my solution with others. MNU 2-01a	<i>I can round a number using an appropriate degree of accuracy, having taken into account the context of the problem.</i> MNU 3-01a	Having investigated the practical impact of inaccuracy and error, I can use my knowledge of tolerance when choosing the required degree of accuracy to make real-life calculations. MNU 4-01a

Estimation & Rounding:

[Estimation for calculations](#)

[Rounding Whole Numbers](#)

[Rounding to Decimal Places](#)

[Rounding to Significant Figures](#)

How this is taught: Estimation for calculations
Outcomes

Using rounding to estimate the answer to a Calculation



We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that our answer is sensible.

Example 1

Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

Estimate = $500 + 200 + 200 + 300 = 1200$

Calculate:

$$\begin{array}{r} 486 \\ 205 \\ 197 \\ +321 \\ \hline 1209 \end{array}$$

Answer = 1209 tickets

Example 2

A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate = $50 \times 40 = 2000\text{g}$

Calculate:

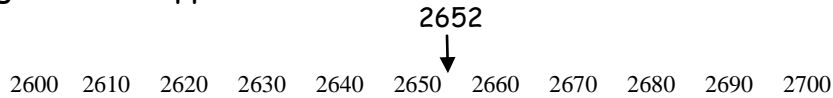
$$\begin{array}{r} 42 \\ \times 48 \\ \hline 336 \\ 1680 \\ \hline 2016 \end{array}$$

Answer = 2016g

How this is taught: Rounding Whole Numbers

Outcomes

Numbers can be rounded to give an approximation. We can either round up or down to get to the approximate value.



2652 rounded to the nearest 10 is 2650

2652 rounded to the nearest 100 is 2700

When rounding numbers which are exactly in the middle, convention is to **round up**.
7865 rounded to the nearest 10 is 7870.



In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example 1 Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

$$\begin{aligned} & 46\ 753 \\ = & \underline{47\ 000} \text{ to the nearest thousand} \end{aligned}$$

How this is taught: Rounding to Decimal Places

Example 1 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

$$\begin{aligned} & 1.57359 \\ & 1.57359 \\ = & \underline{1.57} \text{ to 2 decimal places} \end{aligned}$$

How this is taught: Rounding to Significant Figures

Outcomes

Numbers can also be rounded to a given number of significant figures.

To start counting the significant figures find the first non-zero number on the left hand side.



Example 1 The attendance at the cup final was 83 577. Round this to two significant figures.

The first significant figure is 8

The second significant figure is 3

8 3 5 7 7
↑ ↑
1st 2nd

We then look at the next number and decide whether to round the 3 up or keep it the same. It is 5 so we round the 3 up to 4:

= 84 000 to 2 significant Figures

The same principle applies to rounding decimal numbers.

Example 2 Round 0.15273 to 2 significant figures

The first significant figure is 1 in the tenths place

The second significant figure is 5 in the hundredths place

0 . 1 5 2 7 3
↑ ↑
1st 2nd

We then look at the next number and decide whether to round the 5 up or keep it the same. It is 2 so we keep the 5 the same

= 0 . 1 5 to 2 significant figures

Key Words:

Mathematical Dictionary (Key words):

Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Equals (=)	Is the same as, makes or has the same amount as.
Estimate	To make an approximate or rough answer, often by rounding.
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
Significant figures	The first non-zero figures in a number which give the most information about the size of the number.
Decimal places	Places to the right of the decimal point. The first number to the right is the first decimal place.

Number & Number Processes:

(including addition, subtraction, multiplication, division and negative numbers)

Experiences & Outcomes:

	Second Level	Third Level	Fourth Level
Whole Numbers	<p>I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value.</p> <p style="text-align: right;">MNU 2-02a</p>		
Number Processes	<p>Having determined which calculations are needed, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others.</p> <p style="text-align: right;">MNU 2-03a</p> <p>I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods.</p> <p style="text-align: right;">MNU 2-03b</p>	<p><i>I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions.</i></p> <p style="text-align: right;">MNU 3-03a</p> <p><i>I can continue to recall number facts quickly and use them accurately when making calculations.</i></p> <p style="text-align: right;">MNU 3-03b</p>	<p>Having recognised similarities between new problems and problems I have solved before, I can carry out the necessary calculations to solve problems set in unfamiliar contexts.</p> <p style="text-align: right;">MNU 4-03a</p>
Negative Numbers	<p>I can show my understanding of how the number line extends to include numbers less than zero and have investigated how these numbers occur and are used.</p> <p style="text-align: right;">MNU 2-04a</p>	<p><i>I can use my understanding of numbers less than zero to solve simple problems in context.</i></p> <p style="text-align: right;">MNU 3-04a</p>	

Number & Number Processes:

Whole Numbers: Pupils are expected to be able to work with decimals & fractions beyond the Second Level.

Number Processes:

<u>Addition:</u>	<u>Mental Strategies</u> <u>Written Methods</u>
<u>Subtraction:</u>	<u>Mental Strategies</u> <u>Written Methods</u>
<u>Multiplication:</u>	<u>Mental Strategies</u> <u>Multiplying by 10 & 100</u>
<u>Division:</u>	<u>Written Methods</u>
<u>Order of Calculation:</u>	<u>BOMDAS</u>

Negative Numbers:

Addition
Subtraction
Multiplication
Division

Common Terms:

How this is taught: Addition

Outcomes

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate $54 + 27$

Method 1 Add tens, then add units, then add together

$$50 + 20 = 70 \qquad 4 + 7 = 11 \qquad 70 + 11 = 81$$

Method 2 Split up number to be added into tens and units and add separately.

$$54 + 20 = 74 \qquad 74 + 7 = 81$$

Method 3 Round up to nearest 10, then subtract

$$54 + 30 = 84 \quad \text{but } 30 \text{ is } 3 \text{ too much so subtract } 3;$$
$$84 - 3 = 81$$

Written Method

When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

$$\begin{array}{r} 3032 \\ +589 \\ \hline 1 \end{array} \quad \rightarrow \quad \begin{array}{r} 3032 \\ +589 \\ \hline 21 \end{array} \quad \rightarrow \quad \begin{array}{r} 3032 \\ +589 \\ \hline 621 \end{array} \quad \rightarrow \quad \begin{array}{r} 3032 \\ +589 \\ \hline 3621 \end{array}$$

1 1 1 1 1 1 1

$2 + 9 = 11$

$3 + 8 + 1 = 12$

$0 + 5 + 1 = 6$

$3 + 0 = 3$

How this is taught: Subtraction

Outcomes



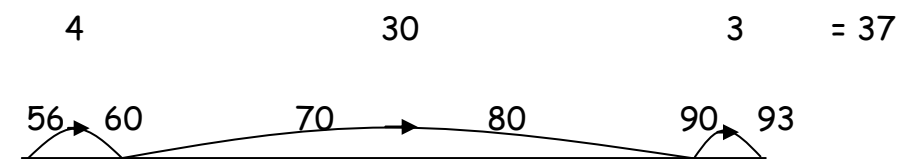
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate $93 - 56$

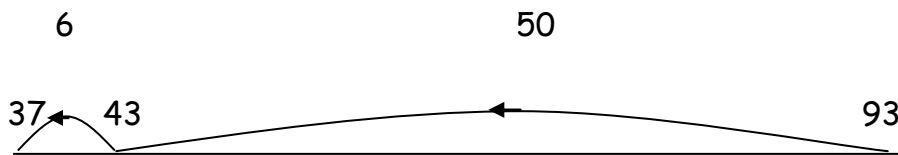
Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 $93 - 50 = 43$
 $43 - 6 = 37$



Written Method

Example 1 $4590 - 386$

$$\begin{array}{r} 4590 \\ - 386 \\ \hline 4204 \end{array}$$

Example 2 Subtract 692 from 14597

We do not "borrow and pay back".

$$\begin{array}{r} 14597 \\ - 692 \\ \hline 13905 \end{array}$$

↑
Start

How this is taught: **Multiplication of Whole Numbers**
Outcomes



It is essential that you know all of the multiplication tables from 1 to 12. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Mental Strategies

Example Find 39×6

Method 1

30×6
 $= 180$

9×6
 $= 54$

$180 + 54$
 $= 234$

Method 2

40×6
 $= 240$

40 is 1 too many
 so take away 6×1

$240 - 6$
 $= 234$

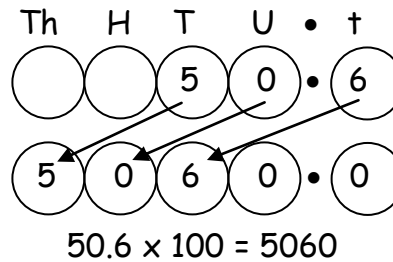
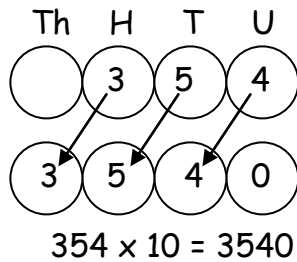
How this is taught: **Multiplication**
Outcomes

Multiplying by multiples of 10 and 100



To multiply by **10** you move every digit *one* place to the left.
 To multiply by **100** you move every digit *two* places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



(c) 35×30

To multiply by 30,
 multiply by 3,
 then by 10.

$35 \times 3 = 105$
 $105 \times 10 = 1050$
 so $35 \times 30 = 1050$

(d) 436×600

To multiply by
 600, multiply by 6,
 then by 100.

$436 \times 6 = 2616$
 $2616 \times 100 = 261600$
 so $436 \times 600 = 261600$



We may also use these rules for multiplying decimal numbers.

For division do the reverse

Example 2 (a) 2.36×20

$2.36 \times 2 = 4.72$
 $4.72 \times 10 = 47.2$
 so $2.36 \times 20 = 47.2$

(b) 38.4×50

$38.4 \times 5 = 192.0$
 $192.0 \times 10 = 1920$
 so $38.4 \times 50 = 1920$

Division

Outcomes



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 24 \\ 8 \overline{) 192} \end{array}$$

There are 32 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r} 1.58 \\ 3 \overline{) 4.74} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0.275 \\ 8 \overline{) 2.2600} \end{array}$$

Each glass contains
0.275 litres

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Order of Calculation (BOMDAS)

Outcomes

Consider this: What is the answer to $2 + 5 \times 8$?

Is it $7 \times 8 = 56$ or $2 + 40 = 42$?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BOMDAS**

The **BOMDAS** rule tells us which operations should be done first.

BOMDAS represents:

(B)rackets

(O)ver (take priority over)

(M)ultiply

(D)ivide

(A)dd

(S)ubtract

Scientific calculators use this rule, some basic calculators may not, so take care in their use.

Example 1 $15 - 12 \div 6$ BOMDAS tells us to divide first
 = $15 - 2$
 = 13

Example 2 $(9 + 5) \times 6$ BOMDAS tells us to work out the
 = 14×6 brackets first
 = 84

Example 3 $18 + 6 \div (5-2)$ Brackets first
 = $18 + 6 \div 3$ Then divide
 = $18 + 2$ Now add
 = 20

Evaluating Formulae:

This will usually be encountered in Maths lessons at first but more able pupils may go on to use these rules when calculating values from formulae at Third & Fourth Levels in a few subject areas.

Outcomes



To find the value of a variable in a formula, we must substitute all of the given values into the formula, then use BOMDAS rules to work out the answer.

Example 1

Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

$$P = 2L + 2B$$

$$P = 2 \times 12 + 2 \times 7$$

$$P = 24 + 14$$

$$P = 38$$

Step 1: write formula

Step 2: substitute numbers for letters

Step 3: start to evaluate (BOMDAS)

Step 4: write answer

Example 2

Use the formula $I = \frac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$

$$I = \frac{V}{R}$$

$$I = \frac{240}{40}$$

$$I = 6$$

Example 3

Use the formula $F = 32 + 1.8C$ to evaluate F when $C = 20$

$$F = 32 + 1.8C$$

$$F = 32 + 1.8 \times 20$$

$$F = 32 + 36$$

$$F = 68$$

Negative Numbers:

Addition & Subtraction:

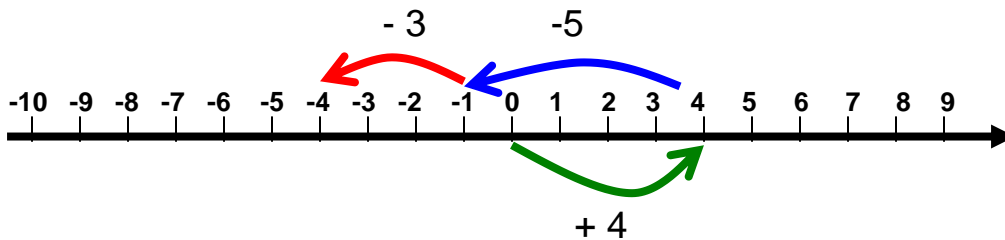
. Using a number line:

To add and subtract numbers always begin counting from zero.

When dealing with positive numbers count to the right. →

When dealing with negative numbers count to the left. ←

e.g. Calculate $4 - 5 - 3$



Start at 0

Move to 4 places to the right (+ 4)

Move 5 places to the left (- 5)

Move 3 places to the left (- 3)

Answer: - 4

When adding or subtracting negative numbers, we must remember that when two signs appear next to each other and are different, then we subtract. When two signs are next to each other and they are the same, we add:

i.e.

$$\begin{array}{l} + + \\ - - \end{array} \left. \vphantom{\begin{array}{l} + + \\ - - \end{array}} \right\} +$$

$$\begin{array}{l} + - \\ - + \end{array} \left. \vphantom{\begin{array}{l} + - \\ - + \end{array}} \right\} -$$

For example:

Calculate:

a) $10 + ^{-} 7$

b) $4 - ^{-} 3$

Solutions:

a) $10 + ^{-} 7 = 10 - 7 = 3$

b) $4 - ^{-} 3 = 4 + 3 = 7$

Multiplying and dividing negative numbers

The rule for multiplying and dividing is very similar to the rule for adding and subtracting. When the signs are different the answer is negative and when the signs are the same the answer is positive:

$+ \times -$	}	-	$+ \times +$	}	+
$- \times +$			$- \times -$		
$+ \div -$			$+ \div +$		
$- \div +$			$- \div -$		

For example:

Calculate:

a) 5×-4

b) $-40 \div -8$

Solution:

a) We have $+5$ and -4 . The signs are different, so the answer will be negative. So, $5 \times -4 = -20$

b) We have -40 and -8 . The signs are the same, so the answer will be positive. So, $-40 \div -8 = 5$

Key Words:

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (\div)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Greater than ($>$)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$
Least	The lowest number in a group (minimum).
Less than ($<$)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.
Maximum	The largest or highest number in a group.
Minimum	The smallest or lowest number in a group.
Minus (-)	Subtract or a negative number
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (\times)	To combine an amount a particular number of times. Example $6 \times 4 = 24$

Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 , 3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BOMDAS (see p9)
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

Fractions, decimal fractions and percentages:

(including ratio and proportion)

Experiences & Outcomes:

Second Level	Third Level	Fourth Level
<p>I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems.</p> <p style="text-align: right;">MNU 2-07a</p>	<p><i>I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations.</i></p> <p style="text-align: right;">MNU 3-07a</p>	<p>I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices.</p> <p style="text-align: right;">MNU 4-07a</p>
<p>I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method.</p> <p style="text-align: right;">MNU 2-07b</p>	<p><i>I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts.</i></p> <p style="text-align: right;">MNU 3-08a</p>	<p>Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real-life problems.</p> <p style="text-align: right;">MNU 4-08a</p>

Fractions

Equivalent fractions

Simplifying fractions

Calculating fractions of a quantity

Decimal Fractions

Percentages

Without a calculator

With a calculator

Finding a percentage

Ratio

Writing ratios

Simplifying ratios

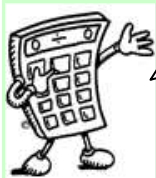
Using ratios

Sharing in a given ratio

Proportion

Fractions 1

Outcomes



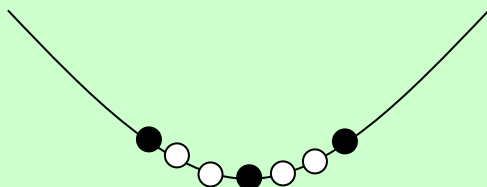
Addition, subtraction, multiplication and division of fractions are studied in mathematics.

However, the examples below may be helpful in all subjects.

Understanding Fractions

Example

A necklace is made from black and white beads.



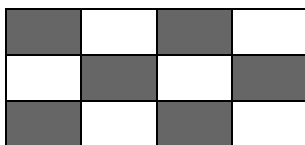
What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example

What fraction of the flag is shaded?



6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

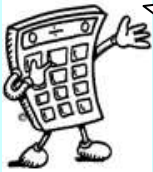
$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Outcomes

Simplifying Fractions

Equivalent fractions can be simplified as shown below:



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.

To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

(a)

$$\frac{20}{25} \begin{array}{c} \div 5 \\ = \\ \div 5 \end{array} \frac{4}{5}$$

(b)

$$\frac{16}{24} \begin{array}{c} \div 8 \\ = \\ \div 8 \end{array} \frac{2}{3}$$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find a **unitary** fraction, **one with a 1 on the top**, of a quantity, divide by the denominator.

To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$$\frac{1}{5} \text{ of } \pounds 150 = \pounds 150 \div 5 = \pounds 30$$

Example 2 Find $\frac{3}{4}$ of 48

$$\frac{1}{4} \text{ of } 48 = 48 \div 4 = 12$$

$$\text{so } \frac{3}{4} \text{ of } 48 = 3 \times 12 = 36$$

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$ **then multiply by**

3

Percentages 1

Outcomes



Percent means out of 100.

A percentage can be converted to an equivalent fraction or decimal.

36% means $\frac{36}{100}$ which equals $36 \div 100$ which equals 0.36

36% = $\frac{36}{100}$ which can be simplified to $\frac{9}{25}$

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
12.5%	$\frac{1}{8}$	0.125
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33\frac{1}{3}\%$	$\frac{1}{3}$	0.333...
50%	$\frac{1}{2}$	0.5
$66\frac{2}{3}\%$	$\frac{2}{3}$	0.666...
75%	$\frac{3}{4}$	0.75

Percentages 2

Outcomes



There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

$$25\% \text{ of } £640 = \frac{1}{4} \text{ of } £640 = £640 \div 4 = £160$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

$$1\% \text{ of } 200\text{g} = \frac{1}{100} \text{ of } 200\text{g} = 200\text{g} \div 100 = 2\text{g}$$

$$\text{so } 9\% \text{ of } 200\text{g} = 9 \times 2\text{g} = 18\text{g}$$

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

$$10\% \text{ of } £35 = \frac{1}{10} \text{ of } £35 = £35 \div 10 = £3.50$$

$$\text{so } 70\% \text{ of } £35 = 7 \times £3.50 = £24.50$$

Percentages 3

Outcomes

Non- Calculator Methods (continued)

The previous 2 methods can be combined so as to calculate any percentage.

Example Find 23% of £15000

$$10\% \text{ of } \pounds 15000 = \pounds 1500 \quad \text{so } 20\% = \pounds 1500 \times 2 = \pounds 3000$$

$$1\% \text{ of } \pounds 15000 = \pounds 150 \quad \text{so } 3\% = \pounds 150 \times 3 = \pounds 450$$

$$23\% \text{ of } \pounds 15000 = \pounds 3000 + \pounds 450 = \pounds 3450$$

Finding VAT (without a calculator)

Value Added Tax (VAT) = 15%

To find VAT, firstly find 10%

Example Calculate the total price of a computer which costs £650 excluding VAT

$$10\% \text{ of } \pounds 650 = \pounds 65 \quad (\text{divide by } 10)$$

$$5\% \text{ of } \pounds 650 = \pounds 32.50 \quad (\text{divide previous answer by } 2)$$

$$\text{so } 15\% \text{ of } \pounds 650 = \pounds 65 + \pounds 32.50 = \pounds 97.50$$

$$\text{Total price} = \pounds 650 + \pounds 97.50 = \pounds 747.50$$

Percentages 4

Outcomes

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

$$23\% = 0.23 \text{ so } 23\% \text{ of } \pounds 15000 = 0.23 \times \pounds 15000 = \pounds 3450$$

$$\text{Or } \pounds 15000 \div 100 \times 23 = \pounds 3450$$



We do not use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals **or finding 1 % and then multiplying.**

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

$$19\% = 0.19 \quad \text{so} \quad \text{Increase} = 0.19 \times \pounds 236000 = \pounds 44840$$

$$\text{Or Increase} = \pounds 236000 \div 100 \times 19 = \pounds 44840$$

$$\begin{aligned} \text{Value at end of year} &= \text{original value} + \text{increase} \\ &= \pounds 236000 + \pounds 44840 \\ &= \pounds 280840 \end{aligned}$$

The new value of the house is £280840

**A more advanced method would be to calculate 119% of £236000, since a 19% increase gives 119%.
e.g. 119% of 236000 = 236000 ÷ 100 × 119 = 280840**

Percentages 5

Outcomes

Expressing something as a percentage



To find a number as a percentage of another number, first make a fraction, this can then be expressed as a percentage by finding that fraction of 100%.

Example 1 There are 30 pupils in Class 3M. 18 are girls. What percentage of Class 3A3 are girls?

$$\frac{18}{30} = 18 \div 30 = 0.6 = 60\%$$

$$\text{OR } \frac{18}{30} \times 100\% = \frac{18 \times 100\%}{30} = \frac{1800}{30} = 60\%$$

60% of 3A3 are girls

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

$$\text{Score} = \frac{36}{44} = 36 \div 44 = 0.81818... = 81.818..% = 82\% \text{ (rounded)}$$

See [Rounding to Decimal Places](#)

$$\text{OR } \frac{36}{44} \times 100\% = \frac{36 \times 100\%}{44} = \frac{3600}{44} = 81.818..% = 82\% \text{ (rounded)}$$

Example 3 In class 2K, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

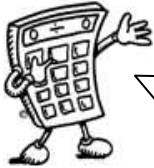
$$\text{Total number of pupils} = 14 + 6 + 3 + 2 = 25$$

6 out of 25 were blonde, so,

$$\frac{6}{25} = 6 \div 25 = 0.24 = 24\% \text{ or same as above}$$

Ratio 1

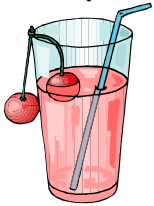
Outcomes



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.

The ratio of water to cordial is 4:1
(said "4 to 1")

The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2



In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

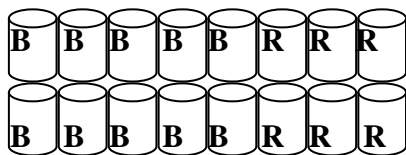
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue : Red = 10 : 6
= 5 : 3

To simplify a ratio, divide each figure in the ratio **by the highest common factor.**

Ratio 2

Outcomes

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6

(b) 24:36

(c) 6:3:12

(a) 4:6
= 2:3

Divide each
figure by 2

(b) 24:36
= 2:3

Divide each
figure by 12

(c) 6:3:12
= 2:1:4

Divide each
figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

$$\begin{aligned}\text{Sand : Cement} &= 20 : 4 \\ &= 5 : 1\end{aligned}$$

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
15	10

Note: In the original image, a bracket on the left side of the Fruit column is labeled 'x5', and a bracket on the right side of the Nuts column is labeled 'x5', indicating that both columns are multiplied by 5 to reach the values 15 and 10.

So the chocolate bar will contain 10g of nuts.

Ratio 3

Outcomes

Sharing in a given ratio

This would be a challenging task at Third Level and more complex examples would be for pupils working at the Fourth Level.

Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$$3 + 2 = 5$$

Step 2 Divide the total by this number to find the value of each part

$$90 \div 5 = \text{£}18$$

Step 3 Multiply each figure by the value of each part

$$3 \times \text{£}18 = \text{£}54$$

$$2 \times \text{£}18 = \text{£}36$$

Step 4 Check that the total is correct

$$\text{£}54 + \text{£}36 = \text{£}90 \quad \checkmark$$

Lauren received £54 and Sean received £36

Proportion

Outcomes

This would be a challenging task at Third Level and more complex examples would be suitable for pupils working at the Fourth Level.



Two quantities are said to be in direct proportion if, when one changes, the other changes in the same way
e.g. if one quantity doubles the other also doubles.

We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1

A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
90	4500

$\left. \begin{array}{c} 30 \\ 90 \end{array} \right\} \times 3$ $\left. \begin{array}{c} 1500 \\ 4500 \end{array} \right\} \times 3$

The factory would produce 4500 cars in 90 days.

Example 2

5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

Find the cost of 1 ticket	→	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">Tickets</td> <td style="padding: 5px; text-align: center;">Cost</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">5</td> <td style="padding: 5px; text-align: center;">£27.50</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">1</td> <td style="padding: 5px; text-align: center;">£5.50</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">8</td> <td style="padding: 5px; text-align: center;">£44.00</td> </tr> </table>	Tickets	Cost	5	£27.50	1	£5.50	8	£44.00	Working:
Tickets	Cost										
5	£27.50										
1	£5.50										
8	£44.00										
		$5 \overline{) £27.50}$	$\begin{array}{r} £5.50 \\ £5.50 \\ \hline 4 \times 8 \\ \hline £44.00 \end{array}$								

The cost of 8 tickets is £44

Key Words:

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$
Calculate	Find the answer to a calculation or problem. It doesn't mean that you must use a calculator!
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (\div)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Greater than ($>$)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$
Least	The lowest number in a group (minimum).
Less than ($<$)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.
Maximum	The largest or highest number in a group.
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.

Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example $6 \times 4 = 24$
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BOMDAS (see p9)
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

Money:

Second Level	Third Level	Fourth Level
<p>I can manage money, compare costs from different retailers, and determine what I can afford to buy. MNU 2-09a</p>	<p><i>When considering how to spend my money, I can source, compare and contrast different contracts and services, discuss their advantages and disadvantages, and explain which offer best value to me.</i> MNU 3-09a</p>	<p>I can discuss and illustrate the facts I need to consider when determining what I can afford, in order to manage credit and debt and lead a responsible lifestyle. MNU 4-09a</p>
<p>I understand the costs, benefits and risks of using bank cards to purchase goods or obtain cash and realise that budgeting is important. MNU 2-09b</p>	<p><i>I can budget effectively, making use of technology and other methods, to manage money and plan for future expenses.</i> MNU 3-09b</p>	<p>I can source information on earnings and deductions and use it when making calculations to determine net income. MNU 4-09b</p>
<p>I can use the terms profit and loss in buying and selling activities and can make simple calculations for this. MNU 2-09c</p>		<p>I can research, compare and contrast a range of personal finance products and, after making calculations, explain my preferred choices. MNU 4-09c</p>

- [Decimal places and money](#)
- [Budgeting](#)
- [Sale Prices & Offers](#)
 - [% Discounts](#)
 - [BOGOF deals](#)
 - [% extra free deals](#)
 - [3 for the price of 2 deals](#)
- VAT [with](#) and [without](#) a calculator
- [Hire Purchase](#)
- [Loans](#)
- [Savings accounts and % Interest rates](#)
- [Interpreting bank statements](#)

Money & Decimal Places

All calculations of money need to be written down to 2 decimal places. This could mean that we need to round numbers:

Example 1 Round £1.525 to 2 decimal places

The second number after the decimal point is a 2 - the check digit (the third number after the decimal point) is a 5, so round up.

$$\begin{array}{r} 1.525 \\ 1.525 \\ = \underline{1.53} \text{ to 2 decimal places} \end{array}$$

We may also need to put in zeros to show our answers to 2 decimal places:

Example 2 Calculate the total cost of the following items

Pencil	20p
Pen	40p
Rubber	30p
Ruler	75p
Sharpener	25p

$$\begin{array}{r} \text{Total cost} = 190\text{p} \\ = \underline{\underline{\pounds 1.90}} \text{ to 2 decimal places} \end{array}$$

Budgeting:

Budgeting is trying to make sure our money lasts until we next receive more money.

To help us to do this we need to have an idea of how much money we will usually receive, our **income**, and how often we will receive it.

We then need to keep track of how much we spend, our **expenditure** or **outgoings**.

From this we can work out how much we have left, our **balance**. To work out the balance we need to subtract our **total expenditure** from our **total income**.

For example: Steven kept a track of his income and expenditure for a month.

Date	Details	Paid In £	Paid out £
15 / 05	Wages	200.00	
17 / 05	CD's R Us		30.00
19 / 05	Sports shop		83.25
22 / 05	Birthday Present		35.00
24 / 05	Sold old CD player on e-bay	25.00	
27 / 05	Cinema		13.50
3 / 06	Bus travel card		39.40
TOTAL:		225.00	201.15

a) Calculate the balance:

$$\begin{array}{rclcl}
 \text{Balance} & = & \text{Total income} & - & \text{Total expenditure} \\
 & = & \text{£ 225.00} & - & \text{£ 201.15} \\
 & = & \underline{\text{£ 23.85}} & &
 \end{array}$$

b) If Steven has a monthly income of £200.00 and typical expenditure of £128, how long will it take for Steven to save up for a new bike at a cost of £135?

$$\begin{array}{rclcl}
 \text{Balance} & = & \text{£ 200} & - & \text{£ 128} \\
 & = & \underline{\text{£ 72}} & &
 \end{array}$$

After 2 months: Balance will be £ 72 x 2 = £ 144

He will be able to afford the bike after 2 months

Sale Prices & Offers:

Percentage Discounts: Non- Calculator Methods



To find the Sale price of an item that has been discounted, first of all we find out the value of the discount. This is the same as finding a percentage **or** **fraction** of an amount.

Method 1 Using Equivalent Fractions

Example A Shirt usually costs £50 but is discounted by 25% in the sale. How much does it cost in the sale?

The shirt has 25% of its value taken away from its original price:

$$25\% \text{ of } £50 = \frac{1}{4} \text{ of } £50 = £50 \div 4 = £12.50$$

Take this away from the original price:

$$£ 50.00 \quad - \quad £12.50 \quad = \quad \underline{£37.50}$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example A bicycle usually costs £200 but is discounted by 20% in the sale. How much does it cost in the sale?

$$1\% \text{ of } £ 200 = \frac{1}{100} \text{ of } £ 200 = £ 2$$

$$\text{so } 20\% \text{ of } £ 200 = 20 \times £ 2 = £ 40$$

Take this away from the original price:

$$£ 200 \quad - \quad £ 40 \quad = \quad \underline{£ 160}$$

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example A pair of football boots are reduced by 30 % in a sale. If their original price was £70, calculate the sale price:

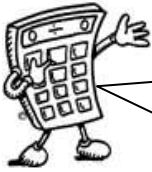
$$10\% \text{ of } £70 = \frac{1}{10} \text{ of } £70 = £70 \div 10 = £7.00$$

$$30\% \text{ of } £70 = 3 \times £7.00 = £21.00$$

Take this away from the original price:

$$£ 70 - £ 21 = \underline{£ 49}$$

Percentage Discounts: Calculator Method



To find a percentage of a quantity using a calculator, change the percentage to a decimal, then multiply or divide by 100, to find 1%, and then multiply by the number of percent you want.

Example 1 A car usually costs £15000 but is reduced by 23% as part of a promotion for this week only. Calculate the cost of the car now.

$$23\% = 0.23$$

$$\text{so } 23\% \text{ of } £15000 = 0.23 \times £15000 = £3450$$

$$\text{or } 23\% \text{ of } £15000 = £15000 \div 100 \times 23 = £3450$$

Take this away from the original price:

$$£ 15\ 000 - £ 3450 = \underline{£ 11\ 550}$$

Buy One Get One Free

This offer is usually used when retailers want to clear a large number of items quickly. They are effectively reducing the price of goods by half whilst ensuring that you buy two items at a time.

This offer is only a saving if you would normally use the two items before the goods would be out of date.

If you usually buy one chocolate cake and you get one free, you haven't made a saving you, just have an extra cake. However, if you usually buy two cakes you have made a saving.

Three for the Price of Two

This is similar to the above offer. The retailers are effectively reducing the cost to two thirds of the original price. This offer is only a saving if you would normally use the three items before the goods would be out of date.

% Extra Free

See [Non-Calculator](#) and [Calculator](#) methods for the different methods of calculating percentages.

Example 1:

A cereal packet usually contains 750g of cereal. There is a special offer packet which contains 25% extra free. How much cereal is in the special offer packet?

Calculate the number of extra grams:

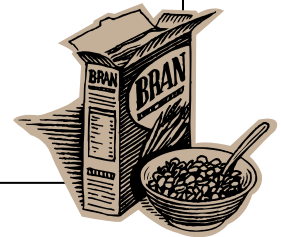
$$25\% \text{ of } 750\text{g} = \frac{1}{4} \text{ of } 750\text{g}$$

$$\frac{1}{4} \text{ of } 750\text{g} = 375\text{g}$$

$$\frac{1}{4} \text{ of } 750\text{g} = 187.5\text{g}$$

Add this to the original number of grams in the packet:

$$750\text{g} + 187.5\text{g} = 937.5\text{g}$$



Which offer is the best value?

To work this out it is useful to compare the products on a price per amount.

Look at the following special offers.

'Swarbricks' 600g 78p per loaf	Brown's Bread 800g £1.20	Wheaty Bake 790g 98p
3 for 2	20% extra free	10% discount

a) Which offers the best value for money per gram of bread without the special offer?

$$\begin{aligned}\text{Swarbricks:} & \quad 78\text{p} \div 600\text{g} \\ & = 0.13\text{p per gram}\end{aligned}$$

$$\begin{aligned}\text{Brown's Bread} & \quad 120\text{p} \div 800\text{g} \\ & = 0.15\text{p per gram}\end{aligned}$$

$$\begin{aligned}\text{Wheaty Bake} & \quad 98\text{p} \div 790\text{g} \\ & = 0.12\text{p per gram}\end{aligned}$$

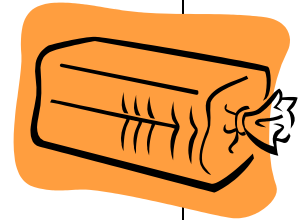
Wheaty Bake is the best value for money at 0.12p per gram

b) Which offers the best value for money per gram of bread with the special offer?

$$\begin{aligned}\text{Swarbricks:} & \quad 3 \text{ for the price of } 2 \\ \text{Cost} & = 2 \times 78 \\ & = 156\text{p} \\ \text{Grams} & = 3 \times 600\text{g} \\ & = 1800\text{g} \\ \\ \text{Cost per gram} & = 156\text{p} \div 1800\text{g} \\ & = \underline{0.09\text{p per gram}}\end{aligned}$$

$$\begin{aligned}
 \text{Brown's Bread} & \quad 20\% \text{ extra free} \\
 \text{Cost} & = 120\text{p} \\
 \text{Grams} = & 800\text{g} + (20\% \text{ of } 800\text{g}) \\
 & = 800\text{g} + 160\text{g} \\
 & = 960\text{g} \\
 \\
 \text{Cost per gram} & = 120\text{p} \div 960\text{g} \\
 & = \underline{0.13\text{p per gram}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Brown's Bread:} & \quad 0\% \text{ extra free} \\
 \text{Cost} & = 98\text{p} - (10\% \text{ of } 98\text{p}) \\
 & = 98\text{p} - 9.8\text{p} \\
 & = 88.2\text{p} \\
 \text{Grams} = & 790\text{g} \\
 \\
 \text{Cost per gram} & = 88\text{p} \div 790\text{g} \\
 & = \underline{0.11\text{p per gram}}
 \end{aligned}$$



The Swarbrick's special offer is the best value for money if you would normally use 3 loaves of bread before the bread went stale.

Finding VAT (without a calculator)

Value Added Tax (VAT) = 15% or **17.5% when rates increase again next year**

To find VAT, firstly find 10%

Example Calculate the total price of a computer which costs £650 excluding VAT

$$\begin{aligned}
 10\% \text{ of } £650 & = £65 && \text{(divide by 10)} \\
 5\% \text{ of } £650 & = £32.50 && \text{(divide previous answer by 2)} \\
 \mathbf{2.5\% \text{ of } £650} & = \mathbf{£16.25} && \text{(divide previous answer by 2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{so } 15\% \text{ of } £650 & = £65 + £32.50 = £97.50 \\
 \mathbf{17.5\% \text{ of } £650} & = \mathbf{£65 + £32.50 + £16.25 = £113.75}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total price (+ 15 \% VAT)} & = £650 + £97.50 = \underline{£747.50} \\
 \mathbf{\text{Total price (+ 17.5 \% VAT)}} & = \mathbf{£650 + £113.75 = \underline{£763.75}}
 \end{aligned}$$

Finding VAT (with a calculator)

Value Added Tax (VAT) = 15%

17.5% when rates increase again next year

Example Calculate the total price of a computer which costs £650 excluding VAT

$$15\% = 0.15$$

$$0.15 \times \text{£}650 = \text{£}97.50 \text{ or } \text{£}650 \div 100 \times 15 = \text{£}97.50$$

$$\text{Total price (+ 15 \% VAT)} = \text{£}650 + \text{£}97.50 = \underline{\text{£}747.50}$$

$$17.5\% = 0.175$$

$$0.175 \times \text{£}650 = \text{£}113.75 \text{ or } \text{£}650 \div 100 \times 17.5 = \text{£}113.75$$

$$\text{Total price (+ 15 \% VAT)} = \text{£}650 + \text{£}113.75 = \underline{\text{£}763.75}$$

What is Hire Purchase?

Hire purchase

A contract to hire goods for a specified period and at a fixed cost. If you pay all the installments over the agreed period, the goods become your property. However, you may return the goods during the specified period, and not be held responsible for paying any future installments. **If you fail to make all the repayments the goods will be taken from you.**

Calculating the cost of a Hire Purchase Agreement

With hire purchase often, a deposit needs to be made. This must be paid before the goods can be taken home,

Often this deposit is a percentage of the cash price .

For Example:

A three piece suite has a Cash price of £1200

Alternatively it can be bought with a 10% deposit and 12 monthly instalments of £99.99

What is the Total cost of the three piece suite with the HP agreement?

$$10\% \text{ of } \text{£}1200 = \text{£}120$$

$$12 \times \text{£}99.99 = \text{£}1199.88$$

$$\text{£}1199.88 + \text{£}120 = \underline{\text{£}1319.88}$$

This agreement costs £119 extra but the customer doesn't need to pay the full price all at once

Loan Agreements

A Loan is a sum of money which is lent out on the proviso that the full amount, plus interest is paid back over a period of time. This is different to an HP agreement in that the item you have bought becomes your property straight away. This is because the total price of the item has been paid out on your behalf by a bank or other money lender.

For example, the same three piece suite can be bought with a £1200 Loan. The bank loaning you the money, say that you must pay the money back as:

6 Repayments of £215

How much more does the loan cost than paying cash?

We can work this out by finding the **total repayments**, and you do this by multiplying 6 by £215.

$$\text{total repayments} = 6 \times \text{£}215 = \underline{\text{£ } 1290}$$

The loan costs £90 more than paying in cash

Savings Accounts and Interest Rates

If you save money in a bank account, the bank will pay you interest. You are, in effect, lending the bank money which they will invest so that the bank can earn more money. They pay you some of this money back as interest.

The amount of interest they will pay is shown as a percentage. The bank will pay you this percentage of the money you have paid in *per year*. This figure is usually called an **APR** or **Annual Percentage Rate**. More recently banks may state the **AER** or **Annual Equivalent Rate**. This rate takes into account how often the bank pays interest.

For Example:

A bank account has an APR of 5% paid once a year. You invest £1000 pounds. How much money will you have at the end of the year?

$$\text{Interest} = 5\% \text{ of } \pounds 1000 = \pounds 1000 \div 100 \times 5$$

$$\text{Interest} = \pounds 50$$

$$\text{Balance at the end of the year} = \pounds 1050$$

The AER is 5%

Another bank account with an APR of 5% will pay 2.5% interest every 6 months. How much money will you have at the end of the year?

After 6 months:

$$\text{Interest} = 2.5\% \text{ of } \pounds 1000 = \pounds 1000 \div 100 \times 2.5$$

$$\text{Interest} = \pounds 25$$

$$\text{Balance at the end of 6 months} = \pounds 1025$$

After a further 6 months:

$$\text{Interest} = 2.5\% \text{ of } \pounds 1025 = \pounds 1025 \div 100 \times 2.5$$

$$\text{Interest} = \pounds 25.63$$

$$\text{Balance at the end of a year} = \pounds 1025 + \pounds 25.63$$

$$= \pounds 1050.63$$

The AER is 5.06%

The higher the APR or AER, the more interest you will earn. Some banks restrict how often you can take your money out of the savings account, especially if the interest rate is high. You should take this into account when deciding which savings account to open.

Understanding Bank Statements

NE Bank Ltd
Statement of account no. 0123456789

Sheet 1

Sort Code: 12-34-56

Date	Type	Details	Paid In £	Paid Out £	Balance £
15 May	CHQ	Wages	200.00		200.00
17 May	DEB	CD's R Us		30.00	170.00
19 May	DEB	Sports Bargain Centre		83.25	86.75
22 May	CPT	Cash Point		35.00	51.75
24 May	CHQ	Cheque no. 123	25.00		76.75
27 May	DEB	Big Screen Cinemas		13.50	63.25
3 Jun	DEB	Transport Executive		39.40	23.85

A bank statement keeps track of the **balance** (how much money is left in the bank account) after every payment or **transaction**. This is shown in the end column.

Every transaction has the date recorded

Banks have codes to show what type of payment has been made.

E.g. CHQ means a cheque

DEB means a debit card

CPT means a withdrawal from a cash point machine

The details can show the name of the company or people money has been paid out to or in from. This can also include cheque numbers and the location of cash machines.

Key Words:

Mathematical Dictionary (Key words):

Estimate	To make an approximate or rough answer, often by rounding.
Discount	The amount of money that the price of an item has been reduced by, the amount taken off the original price.
Regular Price	The original price that an item has been advertised for before a special offer or discount has been.
Sale Price	The new price an item costs after a discount or special offer.
Percentage off	The percentage of the original price that has been taken off.
Bargain	An item that has been bought at a reduced price which the customer believes to be a good deal.
Deals	Another term for a special offer.

Time:

Second Level	Third Level	Fourth Level
<p>I can use and interpret electronic and paper-based timetables and schedules to plan events and activities, and make time calculations as part of my planning. MNU 2-10a</p> <p>I can carry out practical tasks and investigations involving timed events and can explain which unit of time would be most appropriate to use. MNU 2-10b</p> <p>Using simple time periods, I can give a good estimate of how long a journey should take, based on my knowledge of the link between time, speed and distance. MNU 2-10c</p>	<p><i>Using simple time periods, I can work out how long a journey will take, the speed traveled at or distance covered, using my knowledge of the link between time, speed and distance.</i> MNU 3-10a</p>	<p>I can research, compare and contrast aspects of time and time management as they impact on me. MNU 4-10a</p> <p>I can use the link between time, speed and distance to carry out related calculations. MNU 4-10b</p>

[Time notation](#)

[Time Periods](#)

[Distance, Speed & Time](#)

[Interpreting Timetables](#)

[Time, Decimals & Graphs](#)

Time Notation



Time may be expressed in 12 or 24 hour notation.

12-hour clock

Time can be displayed on a clock face, or digital clock.



05:15

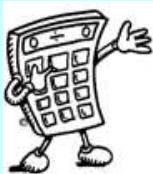
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.

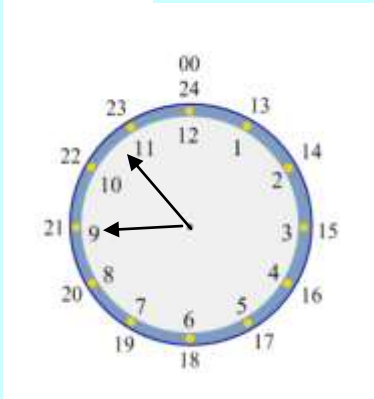
a.m. is used for times between midnight and 12 noon (morning)

p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 ... etc.



Examples

9.55 am → 09 55 hours

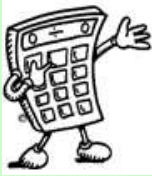
3.35 pm → 15 35 hours

12.20 am → 00 20 hours

02 16 hours → 2.16 am

20 45 hours → 8.45 pm

Time Periods



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)

52 weeks

12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

Distance, Speed and Time.

For any given journey, the distance travelled depends on the speed and the time taken. The relationship between each of these is shown by the following formula:

$$\text{Distance} = \text{Speed} \times \text{Time} \quad \text{or} \quad D = S T$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$

Example 1

Carl travels 70 km in 2 hours. What is his average speed?

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Speed} = \frac{70}{2} = 35 \text{ km/h}$$

Remember: We were given the distance in km and the time in h, so the units for speed are **km/h**.

Interpreting Timetables

Destination	Time	Time	Time	Time	Time	Time	Time	Time	Time
Stornoway Bus Stn.	0755	1030	1300	1430	1540p	1715	1800	1930	2140
W I Hospital	0800	1035v	1305v	1435v	1550p	1720v	1805	1935v	2145v
Barvas Junction	0815	1050	1320	1450	1605	1735	1820	1950	2200+
Upper Barvas	0817	1052	1322	1452	1607	1737	1822	1952	2202
Ballantrushal	0820	1055	1325	1455	1610	1740	1825	1954	2204
Airidhantuim School	0822	1057	1327	1457c	1612	1742	1827	1955	2205
Borve Church	0826	1102	1332	1502c	1617	1747	1832	2002	2212
Melbost Borve	0828	1104	1334	1504c	1619	1749	1834	2004	2214
South Galson	0833	1107	1337	1507c	1622	1752	1837	2007	2217

Examples of Questions:

- a) I want to go to the youth club at Borve Church which starts at 6pm. What time must I catch the bus at Stornoway Bus Station?

6pm is shown as 1800 h on the timetable

The most suitable bus arrives at Borve Church at 1747

This leaves Stornoway at 1715 h

- b) The 0755 bus from Stornoway is running 6 minutes late. What time does it reach South Galson?

Add 6 minutes to the arrival time at Galson

This is 0833 h. It arrives at 0838 h

- c) How long does the first bus journey from W I Hospital to Melbost Borve take?

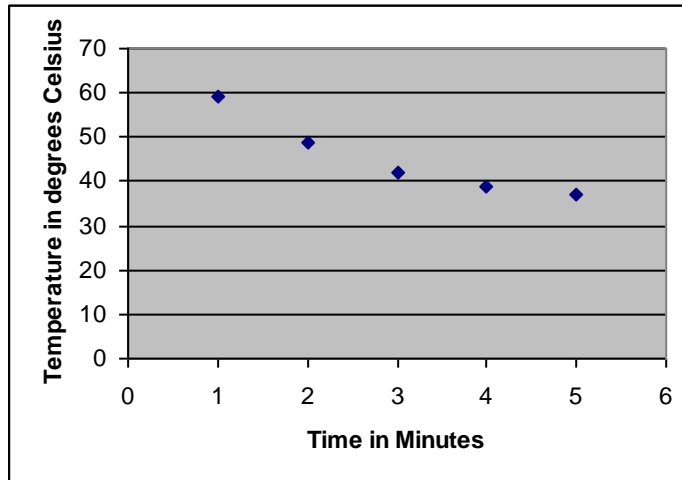
The bus leaves WI Hospital at 0800 h and arrives at Melbost Borve at 0828 h.

The journey time is 28 minutes.

Time, Decimals and Graphs

Pupils can find expressing times as fractions and decimal fractions confusing, leading to errors. This can also cause problems when pupils draw graph scales involving time intervals.

e.g. a pupil may label the axis of a graph as shown.



Pupils sometimes make the error that 50 seconds is equivalent to 0.5 minutes. To avoid this error, pupils should convert minutes into seconds for graph work.

Key Words:

Mathematical Dictionary (Key words):

a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon). am = After midnight
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight). pm = past midday
timetable	

Measurement

Second Level	Third Level	Fourth Level
<p>I can use my knowledge of the sizes of familiar objects or places to assist me when making an estimate of measure. MNU 2-11a</p> <p>I can use the common units of measure, convert between related units of the metric system and carry out calculations when solving problems. MNU 2-11b</p> <p>I can explain how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object. MNU 2-11c</p>	<p><i>I can solve practical problems by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task and using a formula to calculate area or volume when required.</i> MNU 3-11a</p>	<p>I can apply my knowledge and understanding of measure to everyday problems and tasks and appreciate the practical importance of accuracy when making calculations. MNU 4-11a</p>

Reading Scales

Conversions between units

Metric

Imperial

Conversion Graphs

Conversion between units of area and volume

Perimeter

Area

Rectangle

Triangle

Compound Shapes

Parallelogram

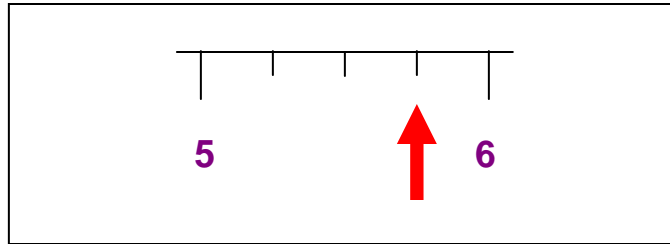
Volume

Cuboids

Other Prisms

Reading scales

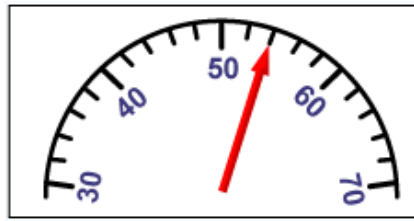
Scale 1



In this scale the difference between 5 and 6 is 1, and the space has been divided into 4, so each division represents $1 \div 4 = 0.25$.

The arrow is pointing to $5 + 0.25 + 0.25 + 0.25 = 5.75$

Scale 2 - a speedometer



The difference between 50 and 60 is 10 and the space has been divided into 5, so each division represents $10 \div 5 = 2$.

The arrow is pointing to $50 + 2 + 2 = 54$.

Converting between units

The table shows some of the most common equivalences between different units of measure. Make sure you know these conversions.

Length	Weight	Capacity
	1 tonne = 1000kg	
1 km = 1000m	1kg = 1000g	
1m = 100cm = 1000mm	1g = 1000mg	1l = 100cl = 1000ml
1cm = 10mm		1cl = 10ml

If converting from a larger unit (eg m) to a smaller unit (eg cm), check what number of smaller units are needed to make 1 larger unit, then **multiply** that number with the relevant number of the larger units.

If converting from a smaller unit (eg cm) to a larger unit (eg m), check what number of smaller units are needed to make 1 larger unit, then **divide** that number into the relevant number of the larger units.

Remember: To convert from a larger unit to a smaller one, **multiply**.
To convert from a smaller unit to a larger one, **divide**.

Worked example

We know that $1\text{m} = 100\text{cm}$

So, to convert from m to cm we multiply by 100, and to convert from cm to m we divide by 100.

Eg $3.2\text{m} = 320\text{cm}$ ($3.2 \times 100 = 320$)
 $400\text{cm} = 4\text{m}$ ($400 \div 100 = 4$)

Metric and imperial units

Imperial measures are old-fashioned units of measure. These days we have mostly replaced them with metric units, but despite our efforts to 'turn metric', we still use many imperial units in our everyday lives. It is therefore important that we are able to calculate rough equivalents between metric and imperial units.

Here are some conversions that you will need to know:

1 inch is about 2.5cm
1 foot is about 30cm
1 gallon is about 4.5 litres

1kg is about 2.2 pounds
1 litre is about 1.75 pints
8km is about 5 miles
(1km is about $\frac{5}{8}$ mile, and 1 mile is about $\frac{8}{5}$ km)

Worked example

We know that 1 litre is about 1.75 pints.

To convert from litres to pints, we multiply by 1.75.

To convert from pints to litres, we divide by 1.75.

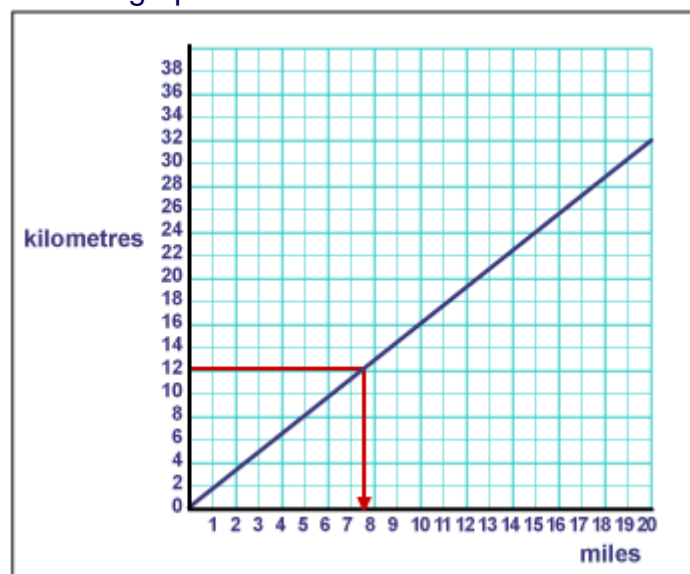
E.g. 2 litres = 3.5 pints ($2 \times 1.75 = 3.5$)

7 pints = 4 litres ($7 \div 1.75 = 4$)

Conversion graphs

Another way of converting units is to use a conversion graph.

The following example shows a graph that enables us to convert between kilometres and miles:



The line on the graph shows that 12km is equivalent to 7.5 miles.

Example:

Use the graph to convert:

a) 20km to miles.

20km is about 12.5 miles.

b) 20 miles to km.

20 miles is about 32km

Converting between area and volume

When you are converting one sort of unit to another, you need to know what number of the smaller units are needed to make 1 larger unit (eg $1000\text{m} = 1\text{km}$), then:

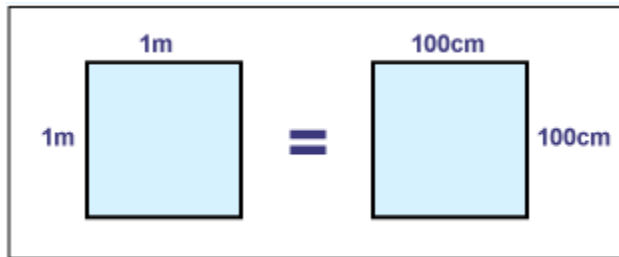
- If converting from a larger unit (eg m) to a smaller unit (eg cm), you **multiply**.
- If converting from a smaller unit (eg cm) to a larger unit (eg mm), you **divide**.

For Example:

Convert 50 000 cm² into m².

Solution

We know that 1m = 100cm.



So, $1\text{m}^2 = 100\text{cm} \times 100\text{cm} = 10\,000\text{cm}^2$.

We are converting from a smaller unit (cm²) to a larger unit (m²), so we **divide**.

$$50\,000\text{ cm}^2 = 50\,000 \div 10\,000 = 5\text{m}^2.$$

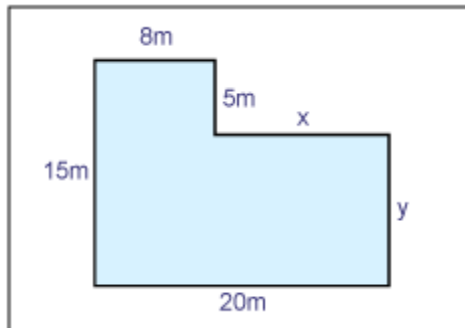
Perimeter

The perimeter of a shape is the length of its boundary.

Think of an ant starting from one corner of a small box, and walking all the way round the edge - what distance will it have walked?

Example question

A plan of a play area is shown below:



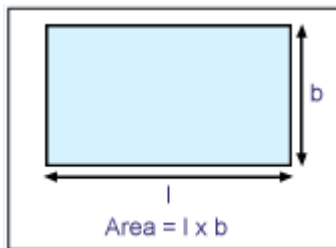
a) Calculate the length of x and y

The length of the play area is 20m, so $x = 20 - 8 = 12\text{m}$. The width of the play area is 15m, so $y = 15 - 5 = 10\text{m}$.

b) Calculate the perimeter of the play area.

$$\begin{aligned}\text{Perimeter} &= 20 + 15 + 8 + 5 + 12 + 10 \\ &= 70 \text{ m}\end{aligned}$$

Area of a rectangle

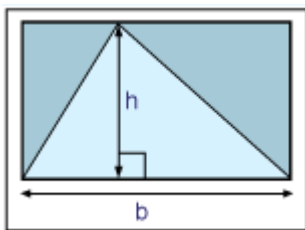


The area of a rectangle is its length multiplied by its breadth.

The formula is: **area = length x breadth**

Area of a triangle

Look at the triangle below:



If you multiplied the base by the perpendicular height, you would obtain the area of a rectangle. The area of the triangle is half the area of the rectangle.

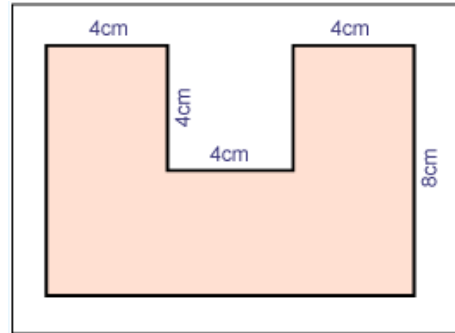
So to find the area of a triangle, we multiply the base by the perpendicular height and divide by two. The formula is:

$$\text{Area} = \frac{b \times h}{2} \text{ or Area} = \frac{1}{2} \times b \times h$$

Area of compound shapes

For example

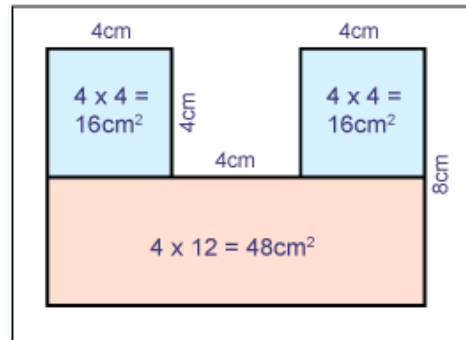
To cover the floor of a doll's house, a carpet is needed with the following shape. Find the area of carpet needed to cover this shape:



Method 1

We can divide the shape into squares and rectangles, find their individual areas and then add them together.

$$\text{Area} = 16 + 16 + 48 = 80\text{cm}^2$$

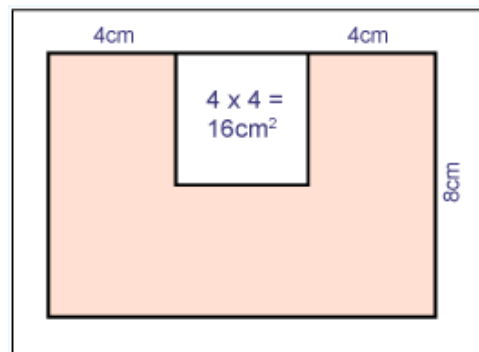


Method 2

We can imagine the shape as a large rectangle with a section cut out!

We find the area of the large rectangle (12 x 8) and then subtract the part that has been cut out (4 x 4)

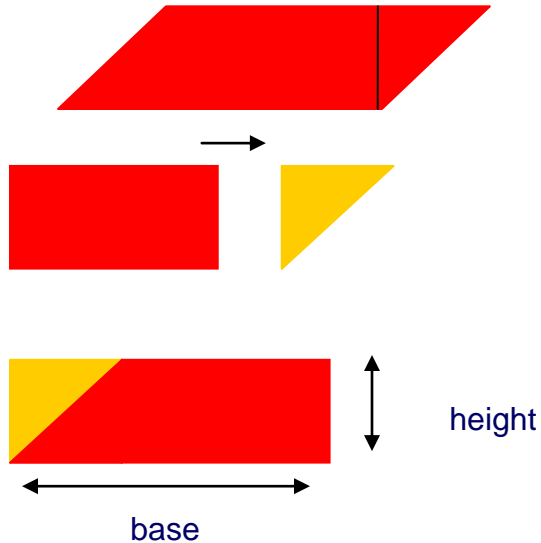
$$\text{Area} = (12 \times 8) - (4 \times 4) = 96 - 16 = 80\text{cm}^2$$



Area of a parallelogram

The area of a parallelogram is $b \times h$ (base \times perpendicular height).

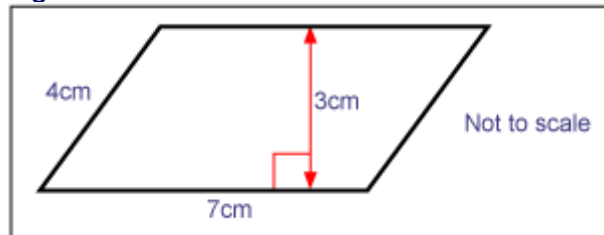
We can see that this is true by rearranging the parallelogram to make a rectangle!



Remember: We are using the perpendicular height of the parallelogram, not the sloping height.

For Example:

Find the area of this parallelogram:



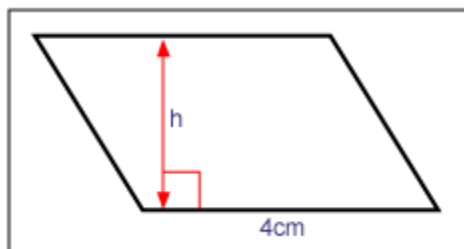
We now know how to find the area of a parallelogram, but what happens if we need to find the base or the height? Easy - we just rearrange the formula!

$$A = b \times h$$

$$h = \frac{A}{b}$$

$$b = \frac{A}{h}$$

For example

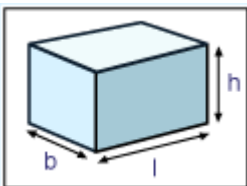


The area of this parallelogram is 12cm^2 , what is its perpendicular height?

Answer

The perpendicular height is $12 \div 4 = 3\text{cm}$.

Volume of Solid Shapes



To find the volume of a cuboid we multiply its length by its breadth by its height.

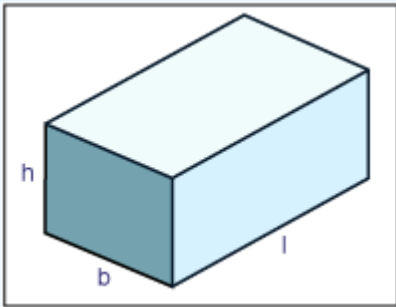
$$\text{Volume} = l \times b \times h$$

For example:

The volume of this cereal packet is $8 \times 20 \times 30 = 4800\text{cm}^3$



Volume of a prism



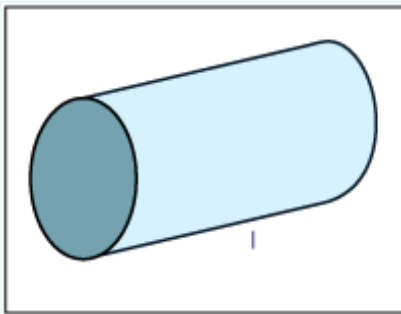
We already know that the volume of a cuboid is $l \times b \times h$.

The area of the shaded end of the cuboid (the cross section) is $b \times h$, so we can also say that the volume of a cuboid is:

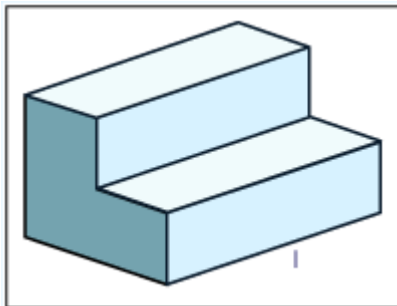
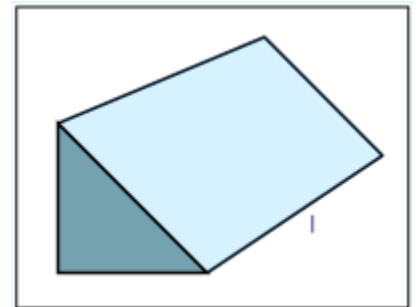
Area of cross section \times length

This formula works for all prisms:

Volume of a cylinder = **area of circle \times length**



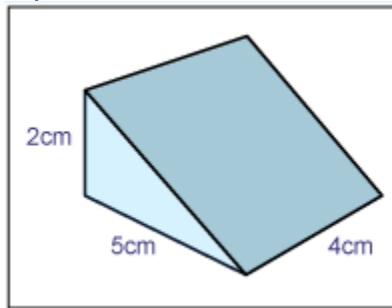
Volume of triangular prism = **area of triangle \times length**



Volume of 'L'-shaped prism = **area of 'L'-shape \times length**

Example 1

What is the volume of this triangular prism?



Answer

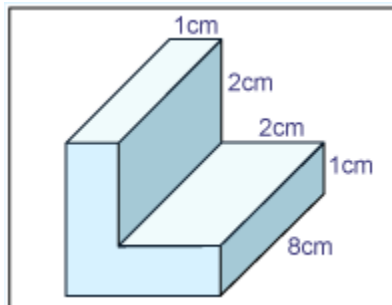
= area of triangle x length

$$\text{Volume} = \frac{1}{2} \times 2 \times 5 \times 4$$

$$= 20\text{cm}^3$$

Example 2

What is the volume of this prism?



Key Words:

Mathematical Dictionary (Key words):

Area	Amount of surface
Perimeter	Distance around the outside edge
Volume	Amount of space inside a shape or the amount of space an object takes up
Sphere	
Prism	3-dimensional shape with the same cross section along its length
Triangular Prism	3-dimensional shape with a triangular cross section along its length
Cylinder	Circular prism - see triangular prism
Cuboid	Rectangular prism - see triangular prism
Cube	
Pyramid	
Cone	

Information handling

Data and analysis

Second Level	Third Level	Fourth Level
<p>Having discussed the variety of ways and range of media used to present data, I can interpret and draw conclusions from the information displayed, recognising that the presentation may be misleading.</p> <p style="text-align: right;">MNU 2-20a</p>	<p><i>I can work collaboratively, making appropriate use of technology, to source information presented in a range of ways, interpret what it conveys and discuss whether I believe the information to be robust, vague or misleading.</i></p> <p style="text-align: right;"><i>MNU 3-20a</i></p>	<p>I can evaluate and interpret raw and graphical data using a variety of methods, comment on relationships I observe within the data and communicate my findings to others.</p> <p style="text-align: right;">MNU 4-20a</p>
<p>I have carried out investigations and surveys, devising and using a variety of methods to gather information and have worked with others to collate, organise and communicate the results in an appropriate way.</p> <p style="text-align: right;">MNU 2-20b</p>		

[Tables](#)

[Bar Graphs](#)

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[Pie Charts](#)

[Averages](#)

[Stem & Leaf Diagrams](#)

Literacy? Press / bias

Conclusions in science

Tables

Outcomes



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	24	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20		2
21 - 25		7
26 - 30		9
31 - 35		5
36 - 40		3
41 - 45		2
46 - 50		2

Each mark is recorded in the table by a tally mark. Tally marks are grouped in 5's to make them easier to read and count.

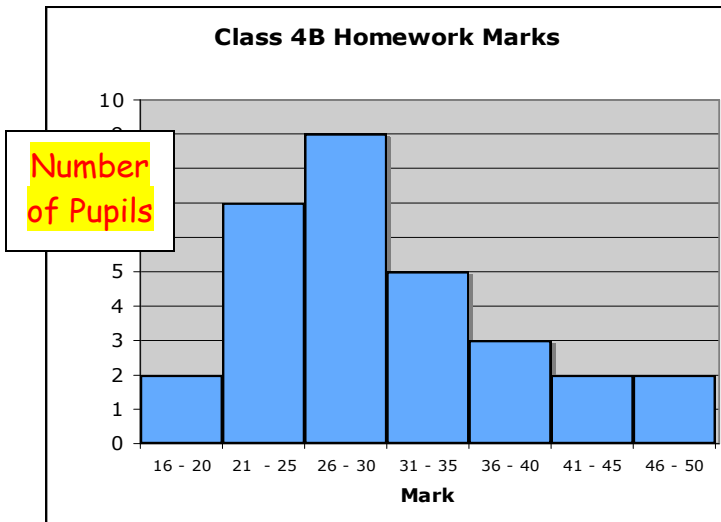
Information Handling : Bar Graphs

Outcomes



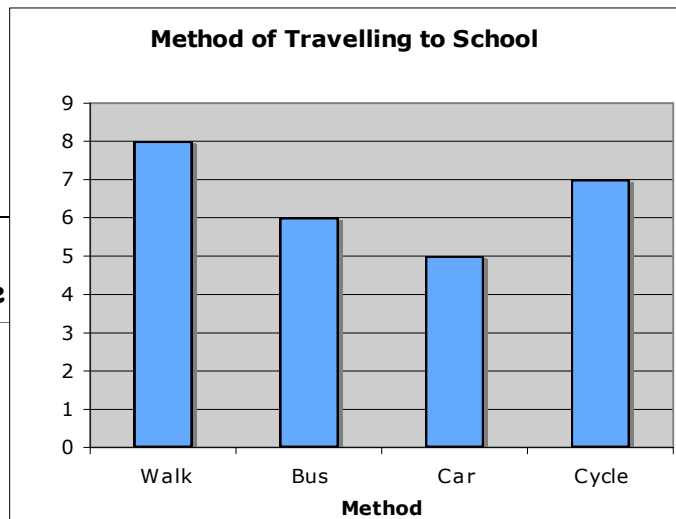
Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 The graph below shows the homework marks for Class 4B.

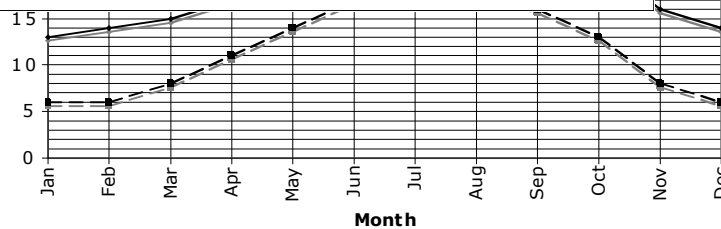


Number of Pupils

Example 2 How do pupils travel to school?



Example



are
d
he

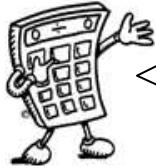
4 weeks as

14

arcelona.

Information Handling : Scatter Graphs

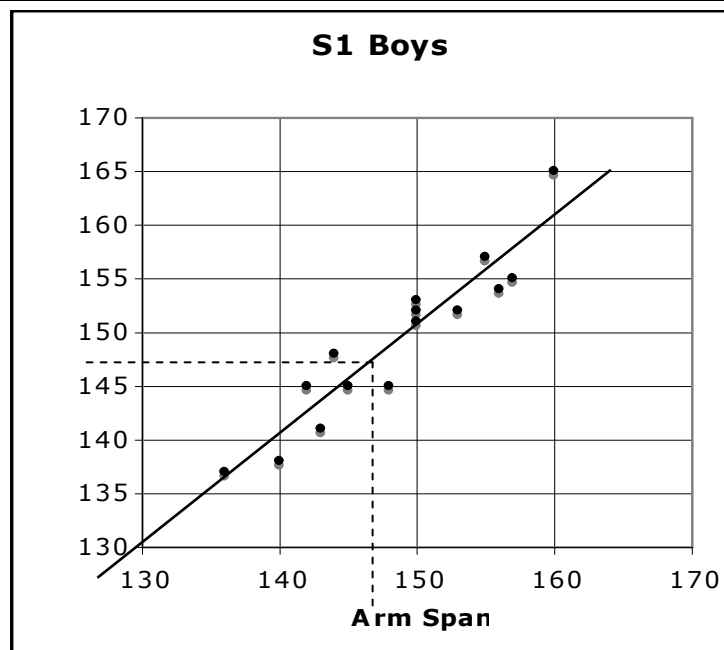
Outcomes



A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137



The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

Note that in some subjects, it is a requirement that the axes start from zero.

Information Handling : Pie Charts

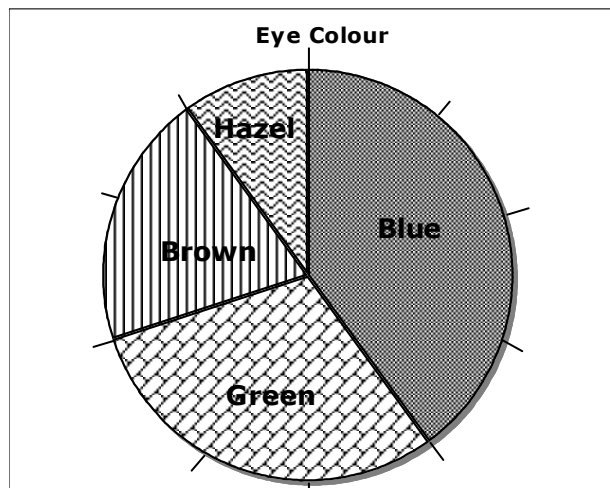
Outcomes



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example

30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72° .

so the number of pupils with brown eyes

$$= \frac{72}{360} \times 30 = 6 \text{ pupils.}$$

If finding all of the values, you can check your answers - the total should be 30 pupils.

Information Handling : Pie Charts 2

Outcomes

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360° .

Example: In a survey about television programmes, a group of people were asked what was their favourite soap. Their answers are given in the table below. Draw a pie chart to illustrate the information.

Soap	Number of people
Eastenders	28
Coronation Street	24
Emmerdale	10
Hollyoaks	12
None	6

Total number of people = 80

$$\text{Eastenders} = \frac{28}{80} \rightarrow \frac{28}{80} \times 360^\circ = 126^\circ$$

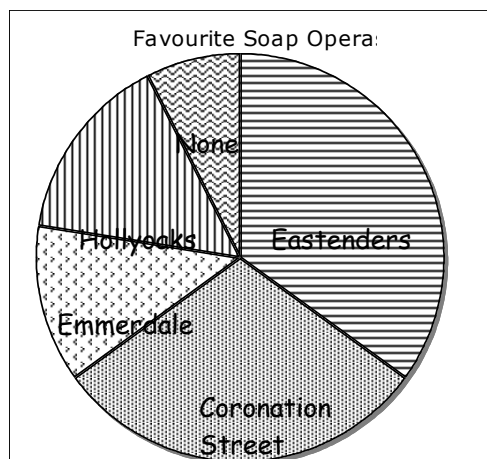
$$\text{Coronation Street} = \frac{24}{80} \rightarrow \frac{24}{80} \times 360^\circ = 108^\circ$$

$$\text{Emmerdale} = \frac{10}{80} \rightarrow \frac{10}{80} \times 360^\circ = 45^\circ$$

$$\text{Hollyoaks} = \frac{12}{80} \rightarrow \frac{12}{80} \times 360^\circ = 54^\circ$$

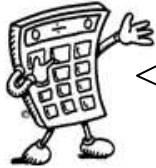
$$\text{None} = \frac{6}{80} \rightarrow \frac{6}{80} \times 360^\circ = 27^\circ$$

Check that the total = 360°



Information Handling : Averages

Outcomes



To provide information about a set of data, the average value may be given. There are **3 different types** of average value - the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value - Lowest value

Example Class 1A4 scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

$$\begin{aligned}\text{Mean} &= \frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14} \\ &= \frac{102}{14} = 7.285\dots \quad \text{Mean} = 7.3 \text{ [to 1 decimal place](#)}\end{aligned}$$

Ordered values: 4, 5, 5, 6, 7, 7, 7, 8, 8, 9, 9, 10, 10
Median = 7

7 is the most frequent mark, so Mode = 7

$$\text{Range} = 10 - 4 = 6$$

Stem and leaf diagrams

A Maths test is marked out of 50.

The marks for the class are shown below:

7	36	41	39	27	21
24	17	24	31	17	13
31	19	8	10	14	45
49	50	45	32	25	17
46	50	23	18	12	6

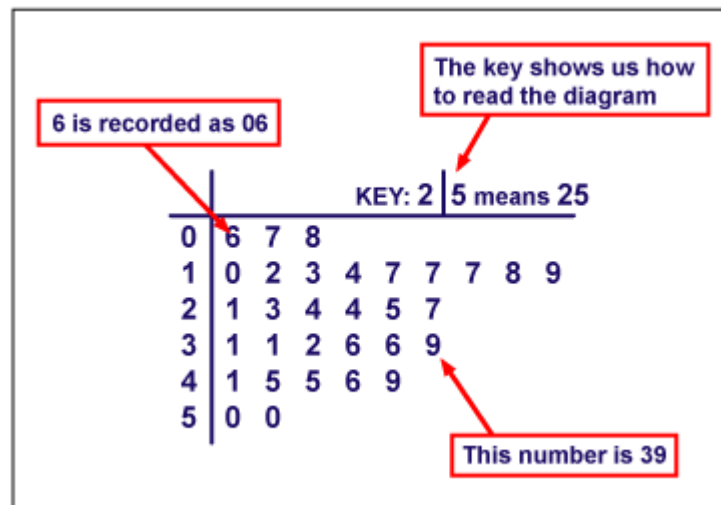
We have all the information we need, but it is very hard to interpret!

For example, is it easy to tell whether more children got marks in the 20s than the 30s?

Can you tell at a glance what the highest mark was, or whether more than one person achieved the same result? - **Probably not!**

One way to overcome this is to represent the data in a more appropriate way. Here we are going to use a **stem and leaf diagram**.

This is an example of a stem and leaf diagram. It shows exactly the same results as in the example above:



The stem and leaf diagram is formed by splitting the numbers into two parts - in this case tens and units.

The tens form the '**stem**' and the units form the '**leaves**'.

This information is given to us in the Key.

It is usual for the numbers to be ordered, so - for example - the row shows the numbers 21,

23, 24, 24, 25, and 27 in order.

2		1	3	4	4	5	7
---	--	---	---	---	---	---	---

Example 1

a) How many children scored 36?

The answer is **two children**
(see right)

		KEY: 2		5 means 25	
0		6	7	8	
1		0	2	3	4 7 7 7 8 9
2		1	3	4	4 5 7
3		1	1	2	6 6 9
4		1	5	5	6 9
5		0			

b) What was the most common score?

The answer is **17 marks**
(see right)

		KEY: 2		5 means 25	
0		6	7	8	
1		0	2	3	4 7 7 7 8 9
2		1	3	4	4 5 7
3		1	1	2	6 6 9
4		1	5	5	6 9
5		0			

Key Words:

Mathematical Dictionary (Key words):

Average	Mean, Median and Mode
Bar Graph	
Data	A collection of information (may include facts, numbers or measurements).
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Line Graph	
Mean	The arithmetic average of a set of numbers (see p32)
Median	Another type of average - the middle number of an ordered set of data (see p32)
Mode	Another type of average - the most frequent number or category (see p32)
Pie Chart	Different ways of presenting data in the form of a graph or chart.
Scatter Graph	
Stem & Leaf Diagram	
Table	

Ideas of chance and uncertainty

Second Level	Third Level	Fourth Level
<p>I can conduct simple experiments involving chance and communicate my predictions and findings using the vocabulary of probability. MNU 2-22a</p>	<p><i>I can find the probability of a simple event happening and explain why the consequences of the event, as well as its probability, should be considered when making choices.</i> MNU 3-22a</p>	<p>By applying my understanding of probability, I can determine how many times I expect an event to occur, and use this information to make predictions, risk assessment, informed choices and decisions. MNU 4-22a</p>

[Describing Probabilities](#)

[Finding Probabilities](#)

[Experimental Probabilities](#)

[Sum of Probabilities](#)

[Combined Events](#)

Relative Frequencies?

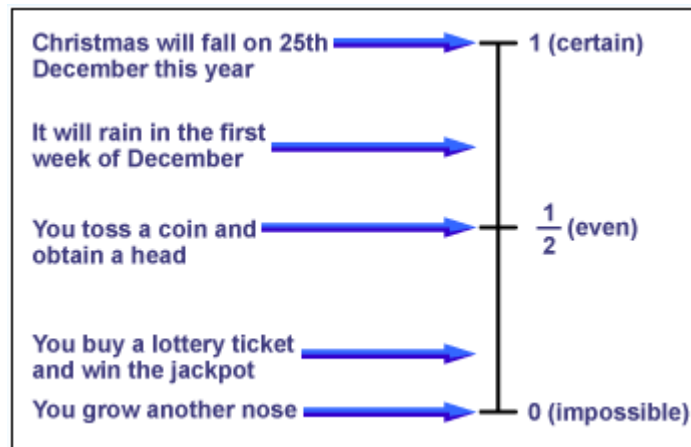
Describing probabilities

We often make judgments as to whether an event will take place, and use words to describe how probable that event is.

For example, we might say that it is likely to be sunny tomorrow, or that it is impossible to find somebody who is more than 3m tall.

The probability scale

In mathematics we use numbers to describe probabilities. Probabilities can be written as fractions, decimals or percentages. We can also use a probability scale, starting at 0 (impossible) and ending at 1 (certain).



Finding probabilities

When we throw a die (plural: dice), there are six possible different outcomes. It can show either 1, 2, 3, 4, 5 or 6.

But how many possible ways are there of obtaining an even number? Clearly, there are three: 2, 4 and 6.

We say that the probability of obtaining an even number is $\frac{3}{6}$
(= $\frac{1}{2}$ or 0.5 or 50%)

The probability of an outcome = $\frac{\text{number of ways the outcome can happen}}{\text{total number of possible outcomes}}$

Example 1

How many outcomes are there for the following experiments?
List all the possible outcomes.

a) Tossing a coin.

There are two possible outcomes (head and tail).

b) Choosing a sweet from a bag containing 1 red, 1 blue, 1 white and 1 black sweet.

There are four possible outcomes (red, blue, white and black).

c) Choosing a day of the week at random.

There are seven possible outcomes (Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday).

Example 2

Sally writes the letters of the word 'MATHEMATICS' on separate cards and places them in a bag. She then draws a card at random.



What is the probability that Sally chooses the letter 'A'?

There are 11 letters in MATHEMATICS, 2 of which are A, so the probability that

$$\text{Sally chooses the letter A is } \frac{2}{11}$$

Experimental probability &

What is wrong with the following statement?

"The probability of obtaining a 6 when I throw a die is $\frac{1}{6}$ - so if I throw the die 6 times I should get exactly **one 6**."

In theory this statement is true, but in practise it is unlikely to be the case. Try throwing a die 6 times - you won't always get **a one**.

Have a go at this:

Example 1:

Kate and Josh each throw a die 30 times.

a) How many times would you expect Kate to obtain a 6?

In theory, Kate should obtain a **6** on $\frac{1}{6}$ of her throws.

$\frac{1}{6}$ of 30 is **5**.

Therefore, we would expect her to obtain a 6 on **5** of her 30 throws.

In theory, Kate should obtain a 6 on $\frac{1}{6}$ of her throws. Therefore, we would expect her to obtain a 6 on 5 of her throws.

b) How many times would you expect Josh to obtain a 6?

We would also expect Josh to obtain a 6 on **5** of his 30 throws.

If an experiment is repeated, the results are not necessarily the same each time.

However, it is more likely that their combined results were closer to the expected outcome (10 x 6s) than their individual results.

In other words, if you do a large number of trials you will get a more accurate result.

Sum of probabilities

If we toss a coin, the probability of obtaining a head is $\frac{1}{2}$ and the probability of obtaining a tail is also $\frac{1}{2}$.

$$P(\text{head}) + P(\text{tail}) = \frac{1}{2} + \frac{1}{2} = 1$$

If we choose a letter at random from the word 'SUMS', the probability of obtaining the letter 'S' is $\frac{2}{4}$, the probability of obtaining the letter 'U' is $\frac{1}{4}$ and the probability of obtaining the letter 'M' is $\frac{1}{4}$.

$$P(S) + P(U) + P(M) = \frac{2}{4} + \frac{1}{4} + \frac{1}{4} = 1$$

We can say that:

'the sum of the probabilities of all possible outcomes is 1'

Example 1

The probability that I am late for work tomorrow is $\frac{2}{9}$. What is the probability that I am not late for work?

Remember that there are two possible outcomes - being late and not being late. The sum of their probabilities must add up to 1, so the probability of not being late is $1 - \frac{2}{9} = \frac{7}{9}$

Combined events

We have seen that **probability** of an outcome is

$$\frac{\text{number of ways an outcome can happen}}{\text{total number of possible outcomes}}$$

However, finding the total number of possible outcomes is not always straightforward - especially when we have more than one event.

Example 1

Two coins are tossed, once each. What is the total number of possible outcomes when they land, with either their heads or their tails uppermost?

Solution

The total number of possible outcomes is not three (two heads, a head and a tail or two tails). To find the true number of possible outcomes, we must list the results or use a table.

1. Using a list

1st coin	2nd coin
H	H
H	T
T	H
T	T

2. Using a table

		1st coin	
		H	T
2nd Coin	H	(H,H)	(H,T)
	T	(T,H)	(T,T)

From both these methods we can see that there are four possible outcomes. We can use this fact to calculate probabilities.

Eg: there is only one way of obtaining two heads, so the probability (P) of obtaining 2 heads is $\frac{1}{4}$.

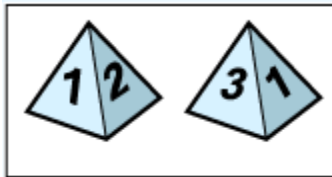
We can say $P(\text{two heads}) = \frac{1}{4}$

There are two ways of obtaining a head and a tail, so $P(\text{head and tail}) = \frac{2}{4} = \frac{1}{2}$

When listing possible outcomes, try to be as logical as possible. If you repeat or forget any of them, it will affect the rest of your answers.

Example question:

Two tetrahedral (four-sided) dice are thrown



Copy and complete the following table, which shows the sum of their scores:

		1st die			
		1	2	3	4
2nd die	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

a) What is the most likely outcome?

5 is the most likely outcome

b) What is the probability that the sum of the scores will be 3?

The probability of getting the sum 3 = $\frac{2}{16} = \frac{1}{8}$

c) What is the probability that the sum of the scores will be greater than 5?

A score greater than 5 occurs 6 times so the probability = $\frac{6}{16} = \frac{3}{8}$

Key Words:

Mathematical Dictionary (Key words):

Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Probability	How likely something is
Outcome	An event that can happen
Possible outcomes	All the possible events that can happen

Glossary of Terms

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (\div)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than ($>$)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$

Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p32)
Median	Another type of average - the middle number of an ordered set of data (see p32)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average - the most frequent number or category (see p32)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example $6 \times 4 = 24$
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BOMDAS (see p9)
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.

Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).